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DISCUSSIONS

APPLICATIONS FOR ADMISSION · AND TRANSFER

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DURATION CURVES

By H. ALDEN FOSTER[1], M. Am. Soc. C. E.

Synopsis

The method of analyzing statistical data by means of the duration curve is discussed in a general way in this paper. The relation of the duration curve to other statistical curves is described. The computation and methods of plotting the curve are outlined, and its mathematical and graphical characteristics are explained and illustrated, followed by a description of graphical methods for using it. Examples of its application to hydro-electric power problems have been included and worked out in detail. The paper concludes with a brief historical review.

Introduction

Of the various graphical methods that have been developed for the study of statistical data, those most familiar to hydraulic engineers are probably the hydrograph, the mass curve, and the duration curve. The first two have been used for many years and are fully described in many textbooks. The duration curve, however, has only been in general use since about 1915. Although it is described in a few recent texts on water-power engineering, there are many matters connected with it which have never been adequately explained in print, as far as the writer has been able to ascertain.

It is the purpose of this paper to describe the general properties of the duration curve, and to present certain applications to hydraulic and water-power problems. The writer does not claim credit for originating the methods of using the duration curve outlined herein, but has tried to give credit for particular methods wherever the source of these methods has been known. His aim has been to describe as briefly as possible the methods used in the office of the firm with which he is associated, with additional comments based on his own study and experience. He has found that those who have not had much experience in the use of duration curves often encounter difficulty through ignorance of the fundamental principles involved. It is hoped that the discussion will bring out additional practical uses of the duration curve.

Note.—Discussion on this paper will be closed in January, 1934, *Proceedings*.

[1] Asst. to Chf. Engr., Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

Graphical Methods for Studying Stream Flow

The simplest method for analyzing stream-flow data is by the hydrograph, Fig. 1(*a*), which shows the monthly run-off of the Cedar River, at Cedar Rapids, Iowa. This is merely a direct graphical plotting of the statistics, the abscissas showing the successive days, and the ordinates the flows occurring on those days. The most important characteristic of the hydrograph is that it presents the statistical data in their true chronological sequence.

The mass curve was first suggested by W. Rippl, in 1882[2]. It is obtained (see Fig. 1(*b*)) by summing the flows of the successive days, weeks, or months, and, hence, gives the total volume of water that has flowed down the stream since the beginning of the record. The mass curve, therefore, is equivalent to the integral of the hydrograph, if the latter could be considered as a continuous mathematical function; and the hydrograph is the derivative of the mass curve. The mass curve represents the area under the hydrograph. The slope of the curve at any point represents the rate of flow at that time. This curve is useful in analyses of stream flow, particularly in studying storage requirements and regulated flows. Its characteristics and uses are well described in many textbooks.

Another method of analyzing the hydrograph, which has long been in use by statisticians, is by the frequency curve, which shows the number of items in the data that have any given magnitude, or the magnitude of which falls within certain limits. Fig. 1(*c*) shows the frequency curve of monthly stream flow on the Cedar River, at Cedar Rapids, for the years 1903 to 1923. Due to the limited number of items, this is plotted as a block diagram rather than as a continuous curve. It shows the number of months in the total period of record (or the frequency of occurrence of the months) in which the stream flow lies between certain limits. For example, during 20 months of the total period, the monthly stream flow was between 3 000 and 3 500 cu ft per sec.

Unless the original data are very numerous, the frequency curve is likely to be rather irregular. Moreover, it does not bear any direct mathematical relation to the hydrograph because the data are not shown in any true chronological order.

If the values shown by the frequency curve are added successively, the "cumulative frequency," or duration curve, is obtained. This curve shows the total number of items in the data which are smaller (or larger) than any given amount. Fig. 1(*d*) gives the duration curve (diagram) corresponding to the frequency diagram of Fig. 1(*c*). It shows the number of months in the period of record in which the flow was equal to or smaller than any given rate. If the total number of time units in the record is taken as 100%, the curve will show the "percentage-of-time" that the flow is equal to or less than any given rate. Such curves therefore, are sometimes referred to as "percentage-of-time," or "%-of-time," curves.

The duration curve is much more regular in shape than the frequency curve, even with only a relatively limited number of items in the data. Since

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[2] *Minutes of Proceedings*, Inst. C. E., Vol. 71, p. 270.

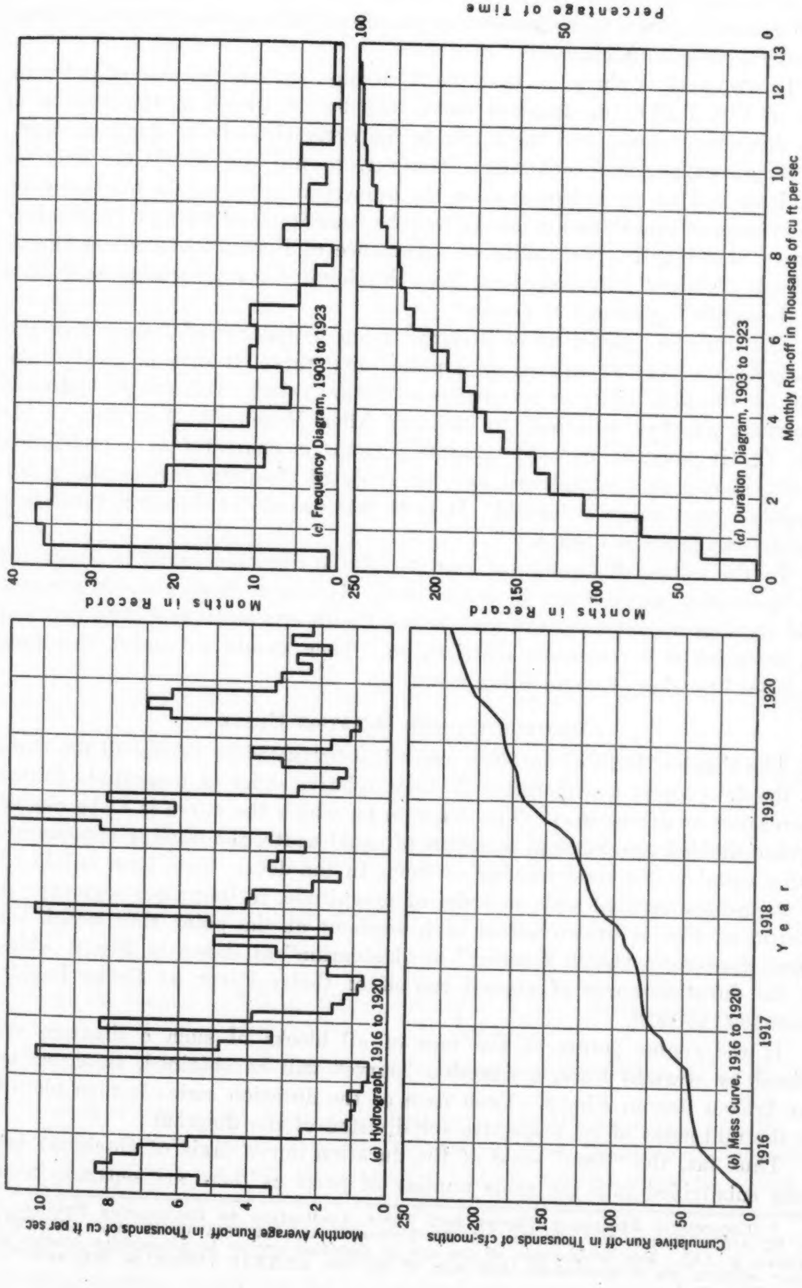

FIG. 1.—MONTHLY AVERAGE RUN-OFF, CEDAR RIVER, AT CEDAR RAPIDS, IOWA.

in most engineering work the data are not very extensive, the duration curve will generally give a more convenient picture of the statistics than is obtained from the frequency curve.

If the curve is placed so that the %-of-time scale is the axis of ordinates (as in Fig. 1(*d*)), the duration curve may be considered as the integral of the frequency curve, and the latter is the derivative of the duration curve. It is generally more convenient, however, to use the %-of-time axis for abscissas and let the ordinates show the numerical values of the original data.

Frequency and duration curves may be considered as forms of probability curves, showing the probability of occurrence of items of any given magnitude in the data. Methods have been developed for representing such data by theoretical equations or curves.[3]

The duration curve, as constructed from actual records, may have two distinct uses: (1) If treated as a probability curve, it may be used to determine the probability of occurrence of future events; this use in hydraulic problems was first proposed[4] by the late Allen Hazen, M. Am. Soc. C. E.; and (2) it can also be used separately from its character as a probability curve, merely as a convenient tool for study of the data, just as the hydrograph or mass curve is treated. It is to this use of the duration curve that the present paper is limited.

In the following discussion, the use of the duration curve is illustrated by application to problems in stream flow. However, these methods are general in their application, and may be used with any statistical data that can be arranged as a frequency distribution. They should be useful, therefore, in other branches of engineering work.

Constructing the Duration Curve

The simplest form of duration curve is constructed by listing all the items in the data under consideration in their relative order of magnitude (either increasing or decreasing). The diagram on which the curve is to be plotted is then divided into vertical segments of equal width, the number of segments being equal to the total number of items in the data. Each item is laid off to the proper vertical scale in order of magnitude, in its proper segment. A horizontal line is drawn across each segment at the point thus noted, the result forming a "block diagram" or "histogram" as shown in Fig. 2, which is the duration curve of annual run-off of Cedar River, at Cedar Rapids, from 1903 to 1923.

If the center points of the tops of all blocks of such a diagram are joined by straight lines, a duration "curve" will be obtained, as shown by the broken line in Fig. 2. Each item of the duration series is then plotted at the mid-point of its respective sub-division of the diagram.

Thus far, the "time" scale of the duration curve (axis of abscissas) has been subdivided into the same number of parts as there are separate items

[3] "Theoretical Frequency Curves and Their Application to Engineering Problems," by H. Alden Foster, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 142. For discussions of the use of duration curves in probability studies by other writers, see *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1921), p. 191, and Vol. 91 (1927), p. 1.

[4] *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 1539.

to be plotted. It is generally more convenient to plot these items on a percentage scale or %-of-time basis. For this purpose the total number of items to be plotted is represented as 100 per cent. If there are n items in

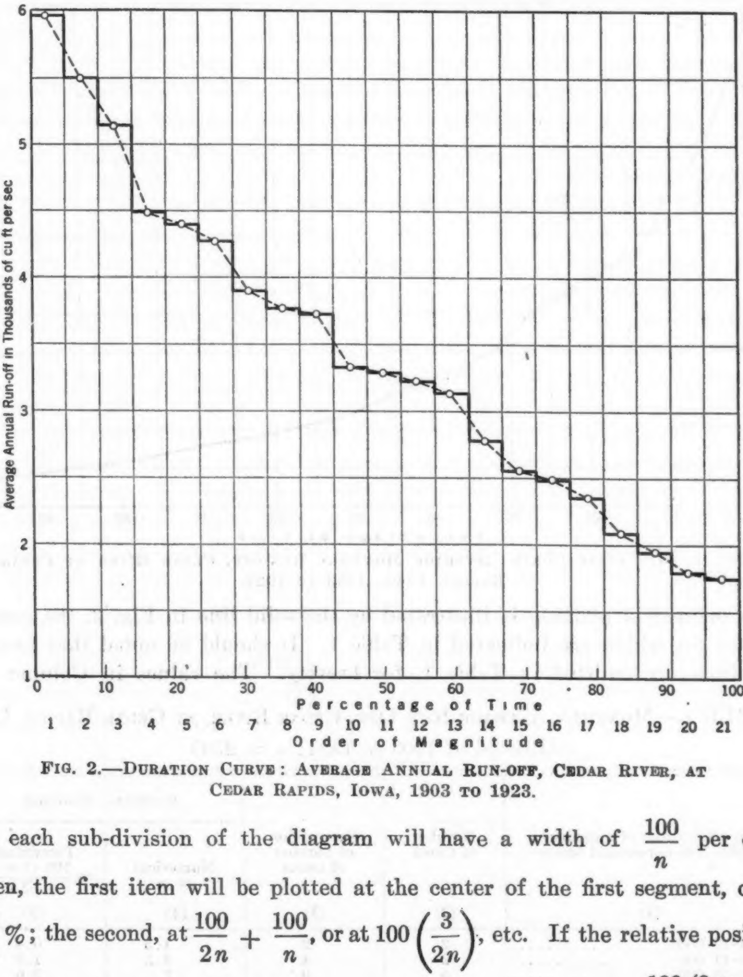

FIG. 2.—DURATION CURVE: AVERAGE ANNUAL RUN-OFF, CEDAR RIVER, AT CEDAR RAPIDS, IOWA, 1903 TO 1923.

all, each sub-division of the diagram will have a width of $\frac{100}{n}$ per cent. Then, the first item will be plotted at the center of the first segment, or at $\frac{100}{2n}$ %; the second, at $\frac{100}{2n} + \frac{100}{n}$, or at $100\left(\frac{3}{2n}\right)$, etc. If the relative position of any item is at the mth segment, its plotting position will be $\frac{100\,(2m-1)}{2n}$ on the %-of-time axis. Such a %-of-time scale is also shown on Fig. 2.

When there are many items in the original data, the labor of tabulating and plotting becomes very great; also, the space between adjacent values on the %-of-time scale becomes too small for accurate plotting. It is then more convenient to group together all items that lie between certain assumed limiting magnitudes, and plot them as one point on the duration curve. The plotting position would then be at the %-of-time corresponding to the middle

item of the group, and its ordinate should equal the average magnitude of the items in the group. If the limits selected for the group are not too far apart, this average may be replaced by the average of the limiting magnitudes.

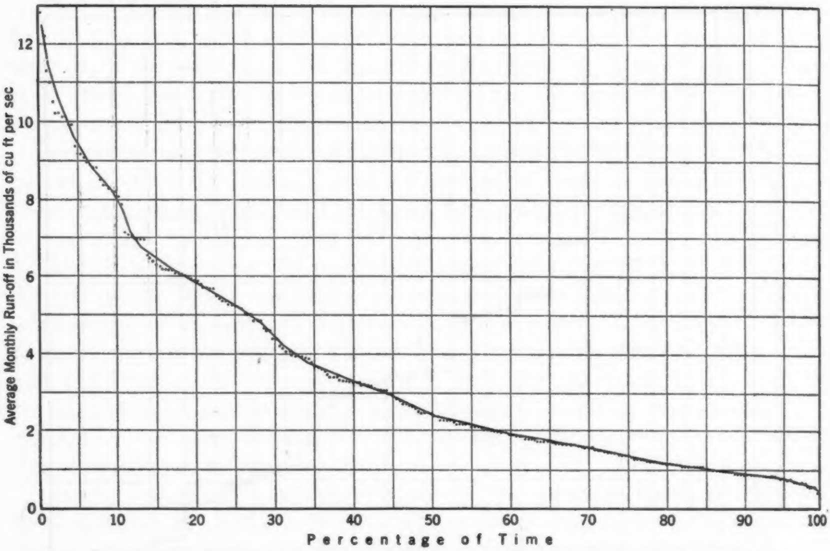

FIG. 3.—DURATION CURVE: AVERAGE MONTHLY RUN-OFF, CEDAR RIVER AT CEDAR RAPIDS, IOWA, 1903 TO 1924.

This method of plotting is illustrated by the solid line in Fig. 3, the computations for which are indicated in Table 1. It should be noted that twenty-two lines are omitted in Table 1, for brevity. The values in Column (4)

TABLE 1.—MONTHLY AVERAGE RUN-OFF, CEDAR RIVER, AT CEDAR RAPIDS, IOWA (Record of 1903 to 1924; $n = 250$)

Run-off in order of magnitude (cubic-feet-per-second limits)	Number of items	Summation of number of items	Plotting Position	
			Numerical $= m$	Percentage $= \frac{100\,(2\,m-1)}{2\,n}$
(1)	(2)	(3)	(4)	(5)
13 000–12 000	2	2	1.5	0.4
12 000–11 000	2	4	3.5	1.2
11 000–10 000	5	9	7	2.6
10 000– 9 000	6	15	12.5	4.8
9 000– 8 500	5	20	18	7.0
******	******	******	******	******
800–700	5	240	238	95.0
700–600	5	245	243	97.0
600–500	4	249	247.5	98.8
500–400	1	250	250	99.8

are obtained by subtracting one-half the number of items in the group from the summation of the number of items and adding 0.5; that is, m equals the numerical order of the center of the group, plus one-half.

Fig. 3 also shows the plotting of the individual monthly values, in order to give a comparison between the two methods of plotting. The method of grouping the items involves only a very slight loss in accuracy if discretion is used in selecting the size of the groups; and the labor involved is greatly reduced.

When individual items are plotted, as shown in Fig. 2, the sum of the plotted ordinates will equal n times the average of all the items. From this it follows that, if the %-of-time method of plotting is used, the total area under the duration curve will equal 100 times the average ordinate, or 100 times the average of the recorded values. Similarly, the area under any part of the curve may be considered to represent the average of the recorded values included in that part.

The method of plotting at the center of each group involves certain principles which should be kept in mind. The only theoretically accurate way to plot a limited number of items is by a block diagram. When an attempt is made to represent the data by a continuous line, it is automatically assumed that there are an infinite number of items to be plotted; that is, from a mathematical point of view, a continuously varying function has an infinite number of possible values. If the data were increased in quantity, there would be several values to plot between each successive pair of items on the original diagram. For example, if in a 5-year record of monthly stream flow there are no recorded values between 2 500 and 3 000 cu ft per sec, one would expect to find one or more monthly flows between these limits if the record were extended to cover 20 or 30 years. A record of infinite length, having an

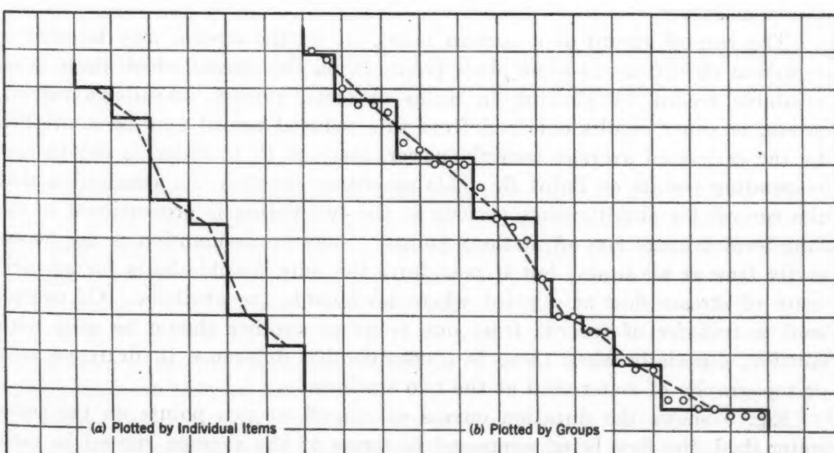

FIG. 4.—TYPICAL SECTIONS OF A DURATION CURVE.

infinite number of items to be plotted, would result in a smooth curve instead of an irregular block diagram. Having only a limited number of items from which to plot a continuous curve, the assumption is made that this continuous curve would pass through the center of each block of the actual record.

When each item of the record is plotted (Fig. 4 (*a*)), the area under the section of the continuous curve within the limits of any part of the diagram will be equal to the corresponding area under the original block diagram. Hence, results obtained from the two diagrams will be in exact agreement. When the items are "bunched" in groups of various sizes, so that the "blocks" of the diagram have variable widths, as shown in Fig. 4(*b*), the curve plotted through the center points of the successive blocks will show a slight discrepancy in area as compared with the original block diagram at any given point; but these errors are compensating. Moreover, the continuous line obtained in this way gives a more satisfactory representation of the complete data than the block diagram obtained by the grouping method.

In general, any method of plotting is satisfactory provided the area under any section of the resulting curve is the same as that under the corresponding section of the block diagram plotted from the complete data.

Plotting in Terms of Mean Flow

When the run-off as recorded at a given point on the stream is to be compared with the record at another point on the same stream, or when the records on different streams are to be compared, it is desirable to divide the original stream-flow values by the average run-off at the point of record, this average being obtained from a record of as great a length as possible. A duration curve constructed from this reduced record will give the flow "in terms of mean run-off," and will reveal as much detail as one obtained from the original data, but in a more convenient form for comparison with other duration curves.

The run-off record at a certain point, *A*, on the stream may be used to represent conditions at some other point, *B*, on the stream where there is no available record, by plotting in terms of mean run-off. Available run-off, power, or other results obtained from this reduced record may be multiplied by the estimated average run-off (power, etc.) at *B*, in order to obtain corresponding results at Point *B*. This procedure involves the assumption that the run-off for simultaneous periods at the two points is proportional to the long-term average run-off at these points. Such an assumption is not necessarily true at all times; but it may form the only feasible basis for an estimate of stream flow at a point where no records are available. Of course, such a transfer of records from one point to another should be used with caution, especially when there is a considerable difference in drainage area or topography of water-shed at the two stations.

Fig. 5 shows the duration curves of run-off at two points on the same water-shed, the flow being expressed in terms of the average run-off in each case. This curve affords an interesting comparison between the stream flow at points of greatly different drainage area and average run-off. The Rogersville record refers to a drainage area of 3 060 sq miles at Rogersville, Tenn., on the Holston River. The average run-off in this case was 4 247 cu ft per sec. The Butler record refers to a drainage area of 427 sq miles at Butler, Tenn., on the Watauga River. The average run-off in this case was

708 cu ft per sec. The length of record is also quite different; but the resulting duration curves show a relatively close agreement as to their general shape, thus indicating that the curve for one point could be used to approxi-

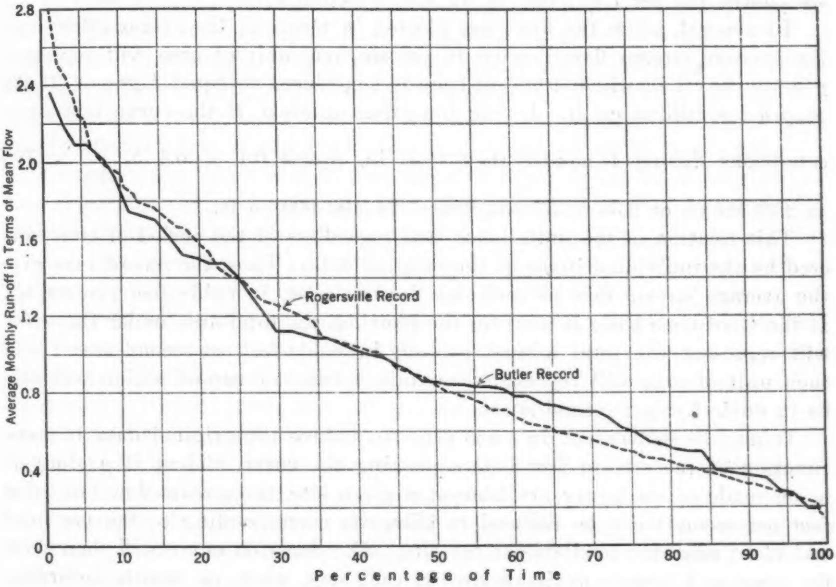

FIG. 5.—DURATION CURVES: AVERAGE MONTHLY RUN-OFF IN TERMS OF MEAN FLOW.

mate conditions at the other point. Such a relationship would be difficult to determine from the original records, due to the great difference in the actual flows at the two points.

UNITS OF MEASUREMENT

With any duration curve of stream flow, since the ordinates represent the rate of stream flow and the abscissas show the time, the area under any part will represent the volume of flow occurring during the periods of time covered by that part; and the total area represents the total volume of run-off during the entire period of record. When the curve is plotted in terms of the mean stream flow and on the %-of-time basis, the total area under the duration curve must equal 100 times the average flow; or, since the average is taken as unity, this area will be 100.

If one linear unit on the horizontal axis represents 100 p %-of-time, and one linear unit on the vertical scale represents q units of stream flow, then one unit of area represents pq volumetric units of run-off. For example, suppose 1 in. on the time scale equals 10% ($p = 0.1$), and 1 in. on the vertical scale equals 0.4 times the average run-off; then 1 sq in. represents 0.04 times the total volume of run-off during the entire period of record. If the duration curve is assumed to represent the conditions during an average year (100%-of-time equals 1 yr), and the average stream flow is 708 cu ft per sec,

then 1 sq in. equals 0.04 × 708 = 28.32 cfs-yr (cubic-feet-per-second-year), or a total volume of 28.32 × 31 536 000 = 893 000 000 cu ft. (A cubic-foot-per-second-year is the total volume produced by a stream flow of 1 cu ft per sec continuing for 1 yr; that is, 31 536 000 cu ft.)

In general, when the flows are plotted in terms of the average flow, and the average stream flow is s cu ft per sec, one unit of area will represent $p\ q\ t\ s$ cfs-yr, in which 100%-of-time is considered to equal t yr—or 31.536 × $p\ q\ t\ s$ million cu ft. In the foregoing example, if the curve represents conditions during 1 month, then 1 sq in. equals $0.1 \times 0.4 \times \frac{1}{12} \times 708 = 2.36$ cfs-yr, or 2.36 × 31 536 000 = 74 400 000 cu ft.

This relation of the units holds true regardless of the period of time covered by the individual items of the original data. Thus, the record may give the average stream flow of each day for one year, in cubic feet per second. If the %-of-time basis is used for the plotting, the total area under the curve will represent the total annual run-off, in cubic-feet-per-second-years; and each unit of area will represent a certain volume of run-off which will also be in cubic-feet-per-second-years.

It may be convenient in some cases to reduce the original data to some function of the stream flow before plotting the curve. Thus, if a study is being made of the power available at a given site, the stream flow (in cubic feet per second) can be reduced to kilowatts corresponding to the net head and plant efficiency available at the site. The duration curve will then show the average kilowatts available during each day, week, or month, according to the time unit used in tabulating the stream-flow data. If the %-of time basis is used for plotting, then one unit of area will equal $p\ q\ t$, in kilowatt-years, in which, one linear unit on the %-of-time axis equals 100 p %, one linear unit on the vertical axis equals q kw, and 100%-of-time equals t yr. If the ordinates give the power in terms of the average kilowatts available (this average being equal to K), and one unit on the vertical axis represents q times the average kilowatts, then one unit of area equals $p\ q\ t\ K$, in kilowatt-years.

In any case, the total area under the curve, when multiplied by the numerical factor corresponding to the scale units used in plotting the curve, should equal the total "output" of the stream during the period of record, in cubic feet, kilowatt-hours, etc., depending on the units in which the data are expressed.

Influence of Time Unit for Original Records

Stream-flow records are generally given in the form of average flow for each successive 24 hours. As has been pointed out, the use of data in this form involves an immense amount of work. It is often convenient to average these data for successive weeks, months, or years. (The United States Geological Survey records are generally given in both daily and monthly form.) Obviously, any lengthening of the time unit used for each item to be plotted in the duration curve will result in a loss of detail, since the flows will not generally be constant over the period of time for which they are averaged. It

is important, therefore, to know what the relative effect will be on the accuracy of the duration curve caused by such changes in the time unit.

In Fig. 6 are shown the daily, monthly, and annual duration curves of stream flow obtained from the same record. These are typical of all duration curves plotted from the same record, but with different time units. As the time unit is increased in length, the lower end of the duration curve will rise and the upper end will be lowered on the diagram. This is to be expected, since the averaging process involved in using a longer time unit must result in eliminating many of the smallest and the largest items in the record.

The total area under each of the curves in Fig. 6 is the same, because this area represents the total volume of run-off during the entire period of record, and this volume is not affected by changing the time unit.

The relative effect of varying the time unit will not be the same with all streams. Where the flow is not subject to any sudden changes, as on the St. Lawrence River, it will be almost constant over considerable periods of time. With such a stream, the daily and weekly duration curves would be almost identical; and the monthly duration curve would not differ greatly from the daily curve. On the other hand, if the stream is "flashy," with sudden floods lasting only a few hours, or days, there will be an appreciable difference between the daily and weekly curves; and the monthly curve will involve a considerable percentage of error as compared with the daily curve.

Unfortunately, it is often necessary to use the monthly instead of the daily or weekly curve, for two reasons: (1) With some streams, only the monthly average flows are available over part or over all of the period of record; and (2) in most preliminary investigations, the time and labor required to prepare the daily or weekly curves are so great that a longer period must be used. In any case, when the monthly curve is used, its limitations should be recognized; and if the stream is at all "flashy," the results should be properly discounted when estimates of plant capacity and output are being made. These comments should apply with equal force to other methods for studying stream-flow data, such as the hydrograph or the mass curve.

The Duration Curve Represents Average Conditions

It is often convenient to consider the duration curve constructed from a given record as representing the average year of that record. Strictly speaking, there is no such thing as an "average year." There may be no particular year in the period of record during which the annual stream run-off, seasonal distribution of flow, maximum and minimum flows, etc., could all be considered as averages of the entire record. The expression is really an abbreviation. When reference is made to the kilowatt-hour output of a hydro-electric plant in an "average year," the meaning is the total number of kilowatt-hours that could be produced during the entire period of record, divided by the number of years of record. This might also be referred to as the "average annual output."

It is in representing such average results that the duration curve is often most useful. The area under any section, if multiplied by the factor, $p\ q\ t$,

will give the volume of run-off represented by that part of the diagram. If the stream flow is given in cubic feet per second, and the number of years of record is t (= 100%-of-time), this volume is expressed in cubic-feet-per-second-years. If this volume is then divided by t, the result will be the average volume per annum (in cubic-feet-per-second-years) represented by that part of the diagram. In other words, the area of a given part of the diagram multiplied by $p\ q$ gives the average annual run-off represented by that part; and the total area under the duration curve represents the average annual run-off of the stream.

The duration curve constructed from a record covering t years not only shows the conditions for all those years, but also may be considered to represent the average annual conditions for the t years. On this basis, it may be stated that the duration curve does represent conditions in the "average year," as indicated by the given record. This is a useful point of view, because it enables the engineer to forget about the length of the original record, and to treat the curve as representing one complete year that is typical of the entire record. Moreover, on the assumption that average results in the future will be similar to those obtained from past records, the same duration curve can also be treated as representing the average year in the future.

In like manner, the duration curve may be considered to represent an average month. In this case, if the ordinates show stream flow, in cubic feet per second, the total area under the curve will represent the average monthly run-off (in cubic-feet-per-second-months), or the total run-off in an "average month."

In some cases, it may be desirable to construct a duration curve for a record of less than one year. Such a curve would be a true representation of conditions for that particular period, but should not be treated as representing conditions for an average year or an average month.

Regulated Flow and Storage

In many problems, it is desirable to construct a duration curve of the flow of a stream (or of some function of the flow, such as the kilowatt capacity of the power house) for conditions after the stream has been regulated by an assumed storage reservoir. The original stream-flow record will furnish a duration curve of "natural flow," before regulation is begun. The same record is then revised to correspond with the assumed method of regulation, and from this is derived a new duration curve of "regulated flow," indicating conditions as they would have existed during the actual period of record if the assumed regulation program had been in effect.

The relation between the duration curves for natural and regulated flow is shown in Fig. 7. The method of regulation illustrated in this diagram is to maintain a constant rate of stream flow (4 000 cu ft per sec) during all periods of low natural run-off, it being assumed that the volume of storage available is sufficient for this purpose. When the natural stream flow is less than the minimum regulated flow, the non-regulated flow is supplemented by enough water taken from the storage reservoir to make the total daily flow equal to the regulated flow. The water taken from the reservoir for this pur-

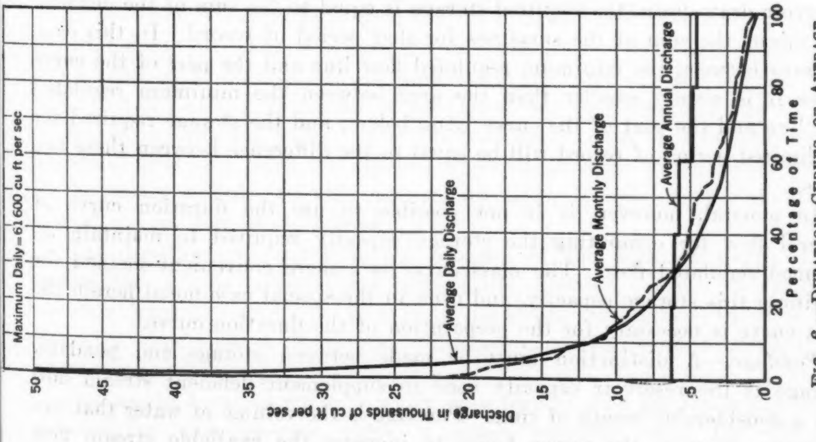

FIG. 6.—DURATION CURVES OF AVERAGE DAILY, MONTHLY, AND ANNUAL DISCHARGE, DELAWARE RIVER, PORT JERVIS, N. Y.; OCTOBER 1, 1916, TO SEPTEMBER 30, 1921.

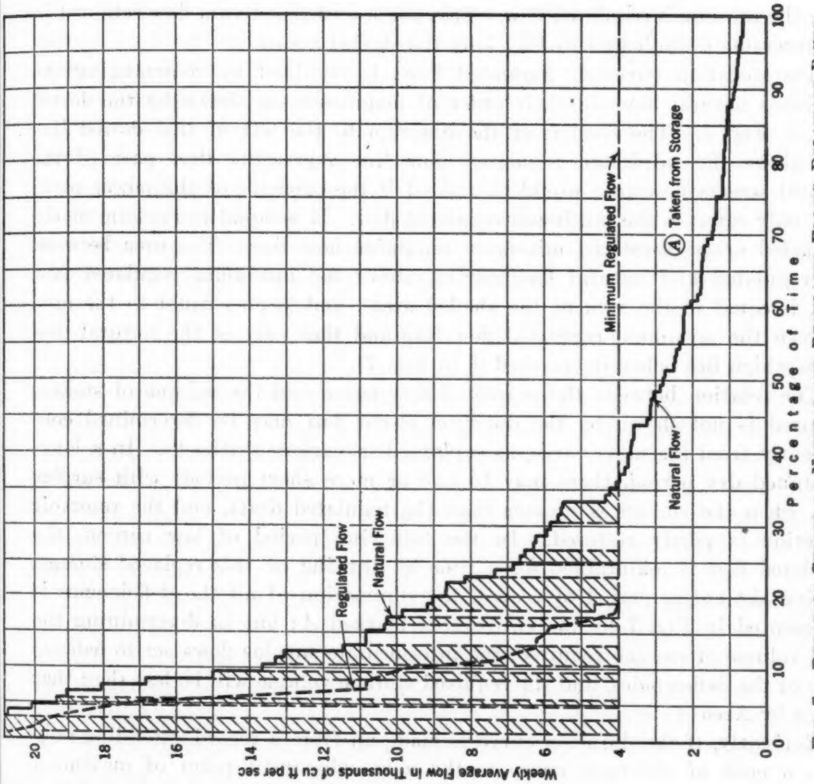

FIG. 7.—DURATION CURVES OF NATURAL AND REGULATED FLOWS, DELAWARE RIVER, PORT JERVIS, N. Y. (WEEKLY FLOW FROM MARCH 29, 1923, TO MARCH 28, 1925, INCLUSIVE).

pose must be replaced during those periods when the natural flow is greater than the minimum regulated flow. This portion of the stream flow retained in the reservoir is indicated in Fig. 7 by the shaded areas.

The duration curve of regulated flows is obtained by re-arranging the corrected natural flows in their order of magnitude, as shown by the dotted line in Fig. 7. The section of the diagram to the left of this dotted line and above the minimum regulated flow line represents that part of the natural stream flow that would be wasted if the capacity of the power plant were only equal to the minimum regulated flow. It is equal to the sum of the unshaded areas above the minimum regulated flow line. The area between the regulated and natural flow curves, above the minimum regulated flow line, is equal to the sum of the shaded areas, and is also equal to the area between the minimum regulated flow line and that part of the natural flow curve which lies below it (marked *A* in Fig. 7).

The relation between the regulated flow curve and the volume of storage required is not shown by the duration curve, but may be determined conveniently from the mass curve, as explained in various textbooks. In a long-continued dry period, there may be one or more short periods with surplus flow, when the run-off is greater than the regulated draft, and the reservoir depletion is partly restored. In the following period of low run-off, the regulated flow is maintained for a time by drawing on this replaced storage. Taking the entire period of record, the summation of all the deficiencies is represented in Fig. 7 by the entire area marked *A*; but in determining the total volume of storage required, the intermediate surplus flows act to balance some of the deficiencies, and the required storage volume will be less than that shown by Area *A*.

Evidently, if the duration curve is made up from a record extending only from a peak of the mass curve to the next subsequent point of maximum reservoir draw-down, the required storage is equal to the sum of the deficiencies minus the sum of the surpluses for that period of record. In this case, the area between the minimum regulated flow line and the part of the curve above it is always smaller than the area between the minimum regulated flow line and the part of the curve lying below; and the storage required for the limited period of record will be equal to the difference between these two areas.[5]

In general, however, it is not possible to use the duration curve of natural flow for computing the storage capacity required to maintain an assumed regulated flow. The mass curve is a more convenient method for obtaining this storage capacity, and even in the special case noted herein the mass curve is necessary for the preparation of the duration curve.

Pondage.—A distinction must be made between storage and pondage. Storage is the reservoir capacity used to supplement deficient stream flow over a considerable length of time. Pondage is the volume of water that can be drawn upon at the power house to increase the available stream flow

[5] "Application of Duration Curves to Hydro-Electric Studies," by G. H. Hickox and G. O. Wessenauer, Juniors, Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., May, 1932, p. 713.

during certain hours of the day. This volume is replaced during the hours when the power load does not require so much water. Storage, only, is considered in determining the regulated flow. The pondage is replaced every twenty-four hours, and is not available for supplementing the natural stream flow during dry periods.

As a special case, part of the pondage may be used for "week-end regulation." The flow of the stream is retained in the pond from Saturday afternoon until Monday morning, and then released on the remaining week days to augment the regulated flow during the hours of maximum load. The effect of pondage on power output is explained in this paper under the heading, "Applications to Hydro-Electric Power Studies."

The Duration-Area Curve

The duration-area curve is a useful device for studies with the duration curve. As far as the writer knows, it was first used by the late E. W. Maloney, M. Am. Soc. C. E. It may be defined as the curve showing the area within the duration curve and beneath a horizontal line having any given ordinate. When the %-of-time basis is used for plotting the curve, such an area also represents the average of the ordinates included in that part of the diagram. Hence, the duration-area curve also shows the average stream flow available below a given draft.

The duration-area curve is illustrated in Fig. 8(*a*). It is conveniently plotted at the left of the sheet, with the scale of ordinates the same as for the duration curve, and abscissas showing the area under the duration curve, or the average ordinate under that curve. (The scale of abscissas is at the top of the diagram.) Thus, in Fig. 8(*a*) the area, *ONKBC*, is represented on the duration-area curve by the abscissa, *NG*; and the average ordinate of this area is 2 500 cu ft per sec.

If the same linear scale is used for the duration-area abscissas as for the duration-curve ordinates, the former curve will intersect the axis of abscissas at an angle of 45 degrees. If the duration curve intersects the 100%-of-time line at an ordinate, y, the duration-area curve will be tangent to this 45° line at the same ordinate value. The duration-area curve will be tangent to a vertical line at a point the abscissa of which is equal to the mean run-off of the entire record, and the ordinate of which is equal to the maximum rate of stream flow in the record.

The duration-area curve is the integral of the duration curve if the stream-flow axis is taken as the base of the duration curve. The slope of the duration-area curve with respect to the stream-flow axis is equal to 0.01 times the %-of time at the corresponding point on the duration curve, if the scales are not distorted.

Just as the duration curve is much more regular than the frequency curve, so also the duration-area curve is much smoother than the duration curve from which it is derived. It can be plotted with considerable accuracy, therefore, from a relatively small number of computed points. In computing

the duration-area curve, the areas under the duration curve can be measured with a planimeter, or the original data may be used.

Approximate Regulated Flow Curve

The actual duration curve of regulated flow can only be determined by a detailed study of the original stream-flow record (perhaps with the assistance of the mass curve) and the construction of a revised record of flow after regulation. This is often a tedious process. An approximate method of constructing such a regulated flow curve has been devised by Mr. E. Laurence Burnett.

This method, as applied to the case of regulation for a constant minimum draft, is illustrated in Fig. 8(*a*). The minimum regulated draft assumed for this example is 2 500 cu ft per sec. The point at which the duration-area curve reaches an abscissa of 2 500 cu ft per sec is noted, as at *G*. The horizontal line, *NG*, is drawn, intersecting the natural flow duration curve at *K*. From the properties of the duration-area curve, the average ordinate of the area, *ONKBC*, is equal to 2 500 cu ft per sec; that is, to the minimum regulated flow, *OP*. Furthermore, the area, *ONKBC*, is equal to the rectangular area, *OPDC*; hence, Area *PNKB* equals Area *BDC*. In other words, if all the surplus flows represented by the area, *PNKB*, are assumed to be stored in the reservoir, they will be sufficient to maintain the regulated draft during all periods of deficient natural flow.

Under this assumed operation, the regulated flow curve would follow the horizontal line, *DM*, as far the %-of-time corresponding to the point, *K*. The remainder of the curve would be found by shifting all that part of the natural-flow curve above *K* vertically downward by the distance, *KM*. The resulting duration curve of "approximate regulated flow" would be the dotted line, *SM*, which is at a constant distance below *AK*.

As a test of the accuracy of this method, the "actual regulated flow" curve is also plotted, as shown by the dashed line. The approximate curve is in fairly close agreement with the actual curve. Fig. 8(*b*) shows another example of this method, in which the approximate curve shows an even closer agreement with the actual curve.

In making computations with the duration curve of regulated flow, it is convenient to have a corresponding duration-area curve of regulated flow. When the approximate method of constructing the regulated flow curve is used, the proper duration-area curve is readily constructed by the following method: The area curve will follow the 45° line, *OT* (Fig. 8(*a*)) up to the ordinate equal to the minimum regulated flow. Beyond this point, it will follow a curve, *TU*, parallel to the original area curve, *GH*, and at the constant distance, *GT*, below the original curve.

In all studies of power output, the regulated-flow duration curve should be used when storage capacity is available for regulating the stream flow. If no storage is available, the natural-flow duration curve should be used. In either case, the proper duration-area curve will be needed, corresponding to the flow-duration curve.

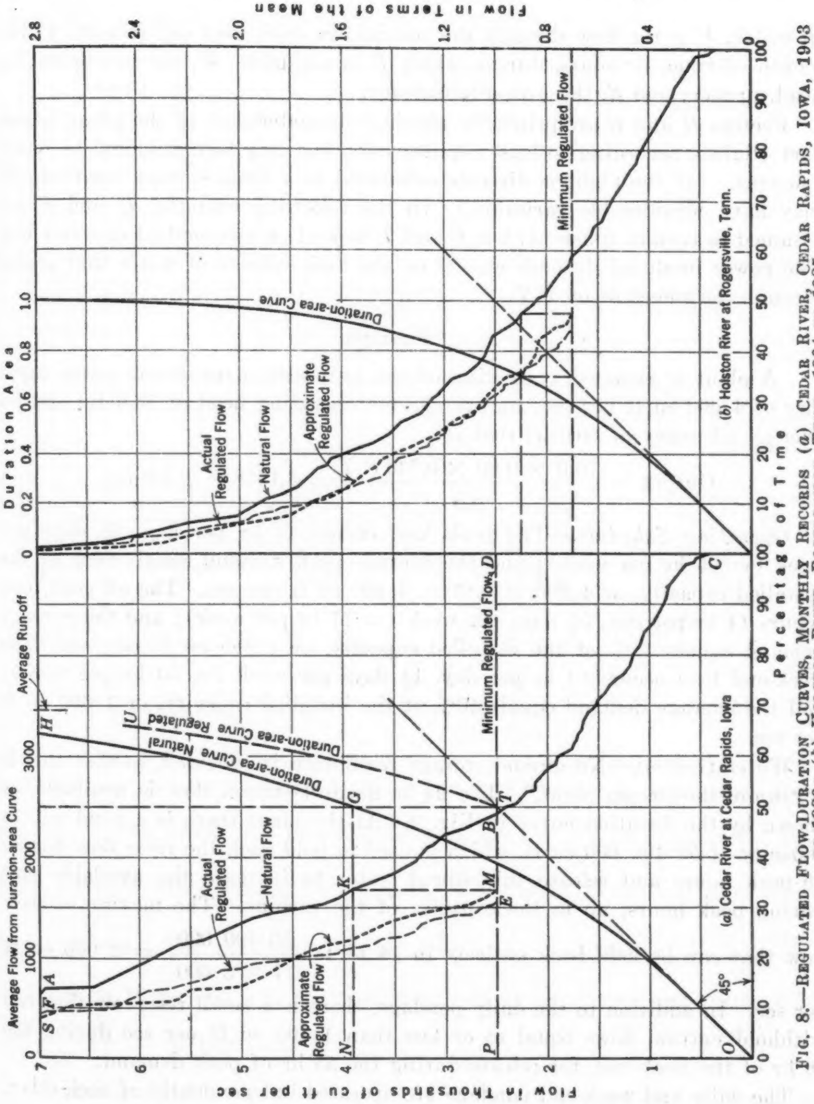

FIG. 8.—REGULATED FLOW-DURATION CURVES, MONTHLY RECORDS (*a*) CEDAR RIVER, CEDAR RAPIDS, IOWA, 1903 TO 1920; (*b*) HOLSTON RIVER, ROGERSVILLE, TENN., 1904 TO 1925.

Application of the Duration Curve to Hydro-Electric Power Studies

The kilowatt-hour output of a hydro-electric plant is equal to $\frac{0.746 \, F \, T \, H \, E}{8.8}$, in which, F is the flow through the turbine, in cubic feet per second; T, the length of time, in hours, during which F is available; H, the net operating head, in feet; and E, the over-all efficiency.

Factors H and E are primarily physical characteristics of the plant, which vary slightly according to load requirements, but may be considered as fairly constant. (If the plant is directly connected to a large storage reservoir, H may have considerable variation.) In the following example, H and E are assumed to remain constant; but F and T depend on the method of water use. The power produced depends closely on the total volume of water that passes through the wheel, or on $F \, T$.

Example

A plant is assumed, consisting of one unit with a maximum water capacity of 4 000 cu ft per sec; an average net operating head of 76.0 ft; and an over-all efficiency of 80.0%; that is:

$$\text{Output} = \frac{76.0 \times 0.80 \times 0.746 \, F \, T}{8.8} = 5.1558 \, F \, T \text{ kw-hr.}$$

Operating Schedule.—The peak load occurs 10 hr per day, $5\frac{1}{2}$ days per week (= 55 hr per week); and the average peak demand equals 85% of the installed capacity, or 4 000 × 0.85 = 3 400 cu ft per sec. The off-peak load occurs 14 hr per day, $5\frac{1}{2}$ days per week (= 77 hr per week); and the average demand equals 45% of the installed capacity, or 1 800 cu ft per sec. The week-end load occurs 24 hr per day, $1\frac{1}{2}$ days per week (= 36 hr per week); and the average demand equals 40% of the installed capacity, or 1 600 cu ft per sec.

Water Control.—No annual storage regulation is provided, so that this is a "run-of-the-stream plant." The 24-hr natural stream flow is available, as shown by the duration curve in Fig. 9. At the plant there is a pond with a capacity of 50 400 000 cu ft, which is used to hold back the river flow during off-peak hours and release this stored water to increase the available flow during peak hours, up to the capacity of the turbine. The maximum river flow that can be held back entirely in 14 hr is $\frac{50\,400\,000}{14 \times 3\,600} = 1\,000$ cu ft per sec. In addition to the daily pondage, there is a small reservoir that can withhold natural flows equal to or less than 1 100 cu ft per sec during the 36 hr of the week-end, for release during the 55 hr of peak demand.

The daily and week-end pondage are operated independently of each other. The cycle of the daily pondage is completed every 24 hr for $5\frac{1}{2}$ days per week, while the cycle of the week-end pondage requires a full week. The peak-power output is produced by the use of the natural stream flow available during the peak hours, plus the additional flow supplied by the daily and week-end pondage.

The duration curve in Fig. 9 shows the distribution of 24-hr average flows throughout a typical year. The same curve may also be considered to indicate the distribution of weekly average flows during the year, without appreciable error, so that any point on the curve represents the average flow

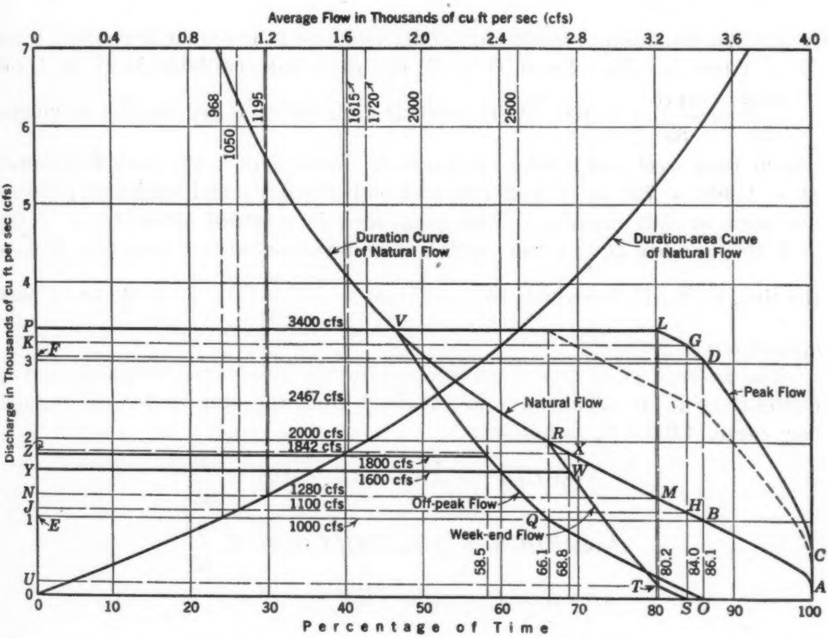

FIG. 9.—EXAMPLE ILLUSTRATING OPERATION OF A HYDRO-ELECTRIC PLANT.

of an entire week. If the natural flow, as shown by the duration curve, is Q cu ft per sec, the week-end pondage can hold back 36 Q cfs-hr when Q is less than 1 100, which will increase the peak use by $\frac{36\,Q}{55}$ cu ft per sec. The daily pondage will increase peak use by $\frac{14\,Q}{10}$, when Q is less than 1 000 cu ft per sec. The maximum benefit from daily pondage is $\frac{14 \times 1\,000}{10}$ = 1 400 cu ft per sec, and the maximum benefit from week-end pondage is $\frac{1\,100 \times 36}{55} = 720$ cu ft per sec.

Off-peak and week-end power will only be produced when the natural flow is greater than is required to refill the ponds during off-peak or week-end periods; that is, when $Q > 1\,000$ cu ft per sec for off-peak power, and when $Q > 1\,100$ cu ft per sec for week-end power.

Average Annual Power Output.—When the natural flow is so small that the daily pondage of 50 400 000 cu ft is not fully used ($Q < 1\,000$ cu ft

per sec), the peak power is produced by a flow of $Q + \frac{14\,Q}{10} + \frac{36\,Q}{55} = \frac{168\,Q}{55}$, in cu ft per sec. This is shown in Fig. 9 by the curve, *C-D*, the ordinates of which are $\frac{168}{55}$ times the corresponding ordinates of the natural flow curve, *A-B*. The maximum benefit from daily pondage is 1 400 cu ft per sec, when $Q = 1\ 000$ (at *B*). From *D* to *G*, the peak flow available is $Q + 1\ 400 + \frac{36\,Q}{55} = \frac{91\,Q}{55} + 1\ 400$. At *G*, when $Q = 1\ 100$ cu ft per sec, the maximum benefit from week-end pondage is reached. From *G* to *L*, the peak flow equals $Q + 1\ 400 + 720 = Q + 2\ 120$, and both the daily and week-end pondage are used at full capacity. Full peak load is supplied when $Q + 2\ 120 = 3\ 400$, or when $Q = 1\ 280$ (at *M*). The duration curve of peak flow, then, is the line, *CDGLP*, for which 100%-of-time $= \frac{55 \times 365}{7} = 2\ 868$ hr of peak operation per year.

The average peak flow is represented by the total area under the curve, *CDGLP* (Fig. 9). This may be obtained from the area under the natural flow curve, *ABHMN*, as follows:

$$\text{Area } CDF = \text{Area } ABE \times 168 \times \frac{1}{55}$$

$$\text{Area } FDGK = \text{Area } EBHJ \times 91 \times \frac{1}{55}$$

$$\text{Area } KGLP = \text{Area } JHMN$$

The areas (in cubic feet per second) under the natural flow curve may be obtained from the duration-area curve. Thus,

$$\text{Area } CDF \quad = 968 \times 168 \times \frac{1}{55} \ldots\ldots\ldots = 2\ 956.8$$

$$\text{Area } FDGK = (1\ 050 - 968) \times 91 \times \frac{1}{55} = \quad 135.7$$

$$\text{Area } KGLP = 1\ 195 - 1\ 050 \ldots\ldots\ldots\ldots = \quad 145.0$$

$$\text{Total} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = 3\ 237.5$$

The average annual peak output equals $3\ 237.5 \times 2\ 868 \times 5.1558 = 47\ 870\ 000$ kw-hr.

Off-peak power is produced when $Q > 1\ 000$ cu ft per sec (at *B*). When the pond is fully emptied each day, 1 000 cu ft per sec must be deducted from the natural flow during off-peak hours to refill the pond, and the flow available for off-peak power will be $Q - 1\ 000$. When Q becomes sufficiently great, the daily pondage will not be fully used. This will occur when the natural flow during peak hours plus $1\ 000 \times \frac{14}{10}$ is greater than peak load, or 3 400;

or when $Q + 1\,400 > 3\,400$, or $Q > 2\,000$ cu ft per sec. Hence, when $1\,000 < Q < 2\,000$, off-peak flow $= Q - 1\,000$. This is represented by the curve OQ. When $2\,000 < Q < 3\,400$, available off-peak flow will equal $\frac{Q \times 24 - 3\,400 \times 10}{14}$, or $\frac{Q \times 24}{14} - 2\,429$. This is shown by the curve, QV; but the average limit of off-peak use is 1 800 cu ft per sec, corresponding to $Q = 2\,467$ cu ft per sec.

The total area (in cubic feet per second) under the off-peak flow curve, OQV, below 1 800 cu ft per sec, is obtained from the natural flow curve, as follows:

$$\text{Area } OQE = \text{Area } EBR2 = 1\,720 - 968 \ldots\ldots = 752$$

$$\text{Area } EQVP \text{ (below 1 800 cu ft per sec)}$$

$$= \frac{(2\,000 - 1\,720)\,24}{14} \ldots\ldots = 480$$

$$\text{Total} \ldots\ldots = 1\,232$$

In this case, 100%-of-time $= \frac{77 \times 365}{7} = 4\,015$ hr per yr of off-peak operation. The average annual off-peak power output is, $1\,232 \times 4\,015 \times 5.1558 = 25\,500\,000$ kw-hr.

Under the assumed method of operation, no week-end power can be produced when $Q < 1\,100$ cu ft per sec. Water for week-end power will equal $Q - 1\,100$, as indicated by the curve, ST, when $1\,100 < Q < 1\,280$. For $Q > 1\,280$, full peak load is supplied, and the week-end pondage will not be fully used. Week-end flow will then equal,

$$\frac{Q \times 168 - 3\,400 \times 55 - (Q - 1\,000)\,77}{36} = \frac{Q \times 91}{36} - 3\,056$$

as shown by the curve, TR. At R ($Q = 2\,000$), the daily pondage is sufficient to carry full peak by itself, and week-end pondage becomes no longer necessary. Above $Q = 2\,000$, the week-end flow $= Q$, and is represented by the curve, RV; but the average limit of week-end use is 1 600 cu ft per sec, and the curve of flows available for week-end power is $STWY$, for which 100% of-time $= \frac{36 \times 365}{7} = 1\,877$ hr per yr. The value of Q corresponding to the point, W, is 1 842 cu ft per sec, as shown at X.

The areas (in cubic feet per second), under the week-end curve are obtained from the natural flow curve, as follows:

$$\text{Area } STU = \text{Area } JHMN = 1\,195 - 1\,050 \ldots = 145.0$$

$$\text{Area } UTWY = \text{Area } NMXZ \times \frac{91}{36}$$

$$= (1\,615 - 1\,195)\,\frac{91}{35} \ldots\ldots = 1\,061.7$$

$$\text{Total} \ldots\ldots = 1\,206.7$$

The average annual week-end power output equals $1\,206.7 \times 1\,877 \times 5.1558 = 11\,680\,000$ kw-hr.

Waste Power.—Since off-peak and week-end loads are less than peak load, there is a waste of possible power within the limit of average peak use (3 400 cu ft per sec). The off-peak waste flow is represented by the area under the line, QVP, and above 1 800 cu ft per sec. This area equals (2 500 $- 2\ 000) \frac{24}{14} = 857$ cu ft per sec. Off-peak waste power $= 857 \times 4\ 015 \times 5.1558 = 17\ 780\ 000$ kw-hr.

Week-end waste flow (in cubic feet per second) is represented by the area, $YWRVP$, which is found as follows:

$$\text{Area } YWR2 = \text{Area } ZXR2 \times \frac{91}{36}$$

$$= (1\ 720 - 1\ 615) \times \frac{91}{36} \ldots\ldots\ldots\ldots = 266$$

$$\text{Area } 2RVP = 2\ 500 - 1\ 720 \ldots\ldots\ldots\ldots\ldots = 780$$

$$\text{Total} \ldots\ldots\ldots\ldots\ldots\ldots\ldots = 1\ 046$$

Week-end waste power equals $1\ 046 \times 1\ 877 \times 5.1558 = 10\ 120\ 000$ kw-hr.

Summary of Annual Power Output.—The total annual power output is as follows:

Class of power	Output, in million kilowatt-hours	Waste, in million kilowatt-hours
Peak power	48.87	0
Off-peak power	25.50	17.78
Week-end power	11.68	10.12
Total	85.05	27.90

The total power possible within the average peak demand (3 400 cu ft per sec) equals $85.05 + 27.90 = 112\ 950\ 000$ kw-hr. As a check on the arithmetical work, the average natural flow below 3 400 cu ft per sec = 2 500 cu ft per sec, and the total possible output equals $2\ 500 \times 8\ 760 \times 5.1558 = 112\ 900\ 000$ kw-hr.

The foregoing example is presented in order to show certain applications of the duration curve to hydro-electric problems. It is intended to be illustrative only, and does not by any means show all the possibilities of the method.

Historical

An attempt has been made to determine the origin of the duration-curve method, but without definite results. Apparently, it has been developed by many engineers, and its conception cannot be credited to any one man, as in the case of the mass curve. In this connection, the following statements[a] by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., are of interest:

"The first printed article that I ever saw on duration curves was about forty or fifty years ago by Clemens Herschel, but I cannot remember the

[a] From a letter dated December 1, 1930.

name of the publication. Somewhere about fifty years ago I had a hand in making a duration curve on a large scale, showing the number of days the Merrimac River at Lawrence had stood above a given height, or a given quantity. This diagram, about 30 inches high by 10 feet long, hung in a prominent place on our office wall and was frequently studied and brought up to date year by year. Probably it hangs there yet with the additions of the past half century. This was made up under the supervision of my chief, Hiram F. Mills."

There are numerous examples of duration curves in various publications subsequent to 1900. The earliest of these is in Mr. Freeman's "Report Upon New York's Water Supply," published in 1900 (page 403). In the 1903 Report of the Commission on Additional Water Supply for the City of New York, the late George C. Whipple, M. Am. Soc. C. E., presented duration curves of the daily flow of the Croton River (Appendix VI, page 486). In the Third Annual Report of the New York State Water Supply Commission, February 1, 1908, Mr. Freeman shows daily and monthly duration curves of the run-off of the Hudson River.

In all these examples, the data are plotted "in order of magnitude," but no use is made of the %-of-time method. This method, however, is used by Walter McCulloh, M. Am. Soc. C. E., in the Sixth Annual Report of the New York State Water Supply Commission, of January 31, 1911, in his report as Consulting Engineer dated December 31, 1910 (pages 119–122). In regard to this report, E. H. Sargent, M. Am. Soc. C. E., writes:

"It is my recollection that this type of curve was original with us, although it may very well be that others entirely independently may have used this sort of curve prior to that time."

The "Introduction to the Theory of Statistics," by G. U. Yule, published in 1910, also gives examples of duration curves which are referred to as "percentile" or "integrated frequency" curves. These are plotted on the %-of-time basis. Since 1910, the %-of-time method of plotting seems to have been in quite general use.

Reference has already been made to Mr. Hazen's paper on "Storage to Be Provided in Impounding Reservoirs." In this paper, the duration-curve method is used with particular regard to its probability features, and a special probability scale is developed for plotting the data. This application of the duration-curve method seems to have been entirely original with Mr. Hazen.[7]

ACKNOWLEDGMENT

The writer wishes to acknowledge the valuable assistance rendered by Mr. E. Laurence Burnett in the preparation of the numerical example included in this paper, and for numerous criticisms and suggestions.

[7] See, also, "Flood Flows," by the late Allen Hazen, M. Am. Soc. C. E., 1930.

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PAPERS

ANALYSIS OF UNSYMMETRICAL CONCRETE ARCHES

By Charles S. Whitney[1], M. Am. Soc. C. E.

Synopsis

In the design of multiple-span arch bridges it is frequently found desirable to raise or lower the abutment end of the end spans to produce the most favorable abutment conditions. The end spans then become unsymmetrical arches which require special analysis.

This paper presents an extension to unsymmetrical arches of the method in the writer's paper[2] entitled, "Design of Symmetrical Concrete Arches," which was based on the work of Strassner.[3] It permits the construction of influence lines for the reactions of an unsymmetrical arch. An attempt is made to clarify the action of the unsymmetrical arch so that each step can be readily understood by the designer.

The simplicity of this method of analysis, which is also quite general in its application, permits a more thorough study of the proportions of an unsymmetrical arch which should result in a more satisfactory design. This paper is concerned only with what has been termed the "geometry" of the problem and does not include the selection of values of modulus of elasticity, movement of abutments, coefficients of thermal expansion, plasticity, shrinkage, etc., although formulas are given to determine their effects on the reactions. The full investigation upon which this paper was based, has been placed on file in Engineering Societies Library, in New York, N. Y.

Introduction

For the purpose of analysis the unsymmetrical arch is divided at the crown point into two parts and the elastic properties of each part are determined separately, after which the action of the entire rib is considered.

Note.—Written discussion on this paper will be closed in January, 1934, *Proceedings*.

[1] Cons. Engr., Milwaukee, Wis.

[2] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 931.

[3] "Neuere Methoden zur Statik der Rahmentragwerke," Vol. II, by A. Strassner, Berlin, Wilhelm Ernst & Sohn.

By using the formulas for arch-axis co-ordinates and rib-thickness variation given in the previous paper,[2] tables have been prepared which give these elastic properties without any involved calculations. In order to avoid lengthy repetition, reference is made to that paper for derivation of basic formulas and a full discussion of their application.

It should be noted that although the tables and diagrams given herein to assist in design are based on certain assumptions regarding the arch-axis curve and the rib-thickness variation, the method of analysis is general and the formulas for reactions can be applied to any arch rib by performing the integrations either graphically or numerically using the actual rib dimensions.

The treatment is arranged in three parts, giving first, the elastic properties of the rib segments to each side of the crown; second, the reactions of the complete rib; and third, a numerical example and a brief discussion of unsymmetrical arch design.

I.—Elastic Properties of Rib Segments

Equation of Arch Axis.—In order to permit an accurate mathematical analysis formulas for the co-ordinates of the arch axis and the variation of moment of inertia along the rib have been derived by the writer in his previous paper entitled "Design of Symmetrical Concrete Arches".[2] The arch-axis formula is a transformed catenary giving a curve that lies very

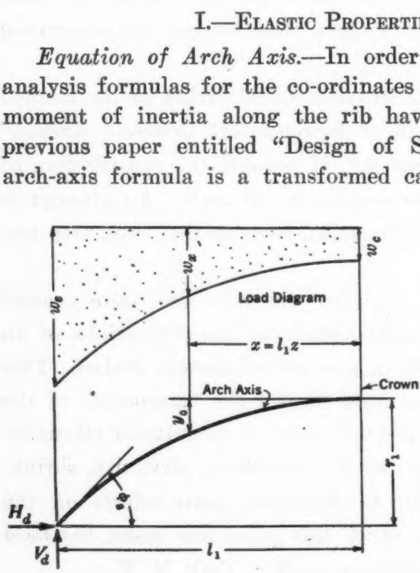

Fig. 1.

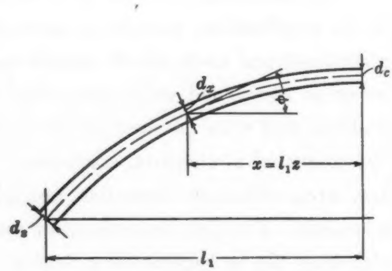

Fig. 2.

close to actual dead load pressure lines. Its form is dependent on the ratio of w_s, the dead load per linear foot at the springing, to w_c, the dead load at the crown. For the special case of $w_s = w_c$, the axis becomes parabolic.

It is necessary to consider concentrated loads as distributed in determining the axis curve, but the effects of the concentration can be determined from the influence lines.[4]

[4] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), pp. 970, 1089–1092, inclusive.

The general formula[5] for the arch axis is (see Fig. 1),

$$y_0 = \frac{r}{g-1} (\cosh z k - 1) \qquad \text{.................(1)}$$

in which, $g = \frac{w_s}{w_c} = \cosh k$; or, $k = \cosh^{-1} g$.

The ordinate, when $x = \frac{l_1}{2}$ (the quarter-point ordinate), is a direct function of g and can be used instead of g to establish the curve if desired. The ratio of this ordinate to the rise, r, is designated as N. When $g = 1$, the curve is a parabola and $N = 0.25$.

The dead load reactions can be computed by the formulas:

$$V_d = w_c\, l_1 \frac{\sqrt{g^2 - 1}}{k} \qquad \text{.....................(2)}$$

and,

$$H_d = w_c\, l_1^2 \frac{(g-1)}{r_1\, k^2} \qquad \text{.....................(3)}$$

in which, the subscript, 1, indicates the left-hand segment of the rib and the subscript, 2, indicates the right-hand segment. It is assumed in all formulas that the proper values of g, k, l_1, l_2, etc., will be used, depending on which segment is being considered. Tables giving the values of $\frac{y_0}{r}$, N, g, H_d, V_d, and tan ϕ_s, will be found in the previous paper.[6]

With the unsymmetrical arch, different curves will be used for the two sides because the dead load at one end will not be the same as that on the other.

If the roadway is nearly level as in most cases, the arch-axis formula can be applied to the two sides independently after the crown point has been located so that the horizontal thrusts are balanced.

If the slope of the roadway is unusually steep it may be best to determine the dead load pressure line graphically from the actual loading and to measure the "quarter-point" ordinates to determine the proper values of N. The curves of the axis on each side can then be established from the respective values of N. The arch-axis curve will then pass through the dead load pressure curve at five points, crown, quarter-points, and springings, and will ordinarily correspond satisfactorily for the full span.

After the rise and loading of the two segments of the arch have been determined, the horizontal position of the crown point can be located by using Equation (3) and placing H_d for the left segment equal to H_d for the right segment.

Variation of Thickness of Rib.—In order to determine the reactions of an arch, it is necessary to assume the relative values of the moments of inertia of the cross-section of the rib at the crown and at the springings. Since the two segments are considered separately, it is not necessary to have the section at the left springing the same as that at the right. The variation in moments of inertia in both segments from crown to springing is assumed

[5] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), Equation (38), p. 968.
[6] *Loc. cit.*, Table 6, p. 973; and Table 9, p. 976.

to follow the relation,[7]

$$I_x = \frac{I_c}{[1 - (1 - m)\, z] \cos \phi} \quad \text{.....................(4)}$$

in which, I_c = the moment of inertia of crown section; I_x = the moment of inertia of section at distance, x; I_s = the moment of inertia at springing; and, $m = \frac{I_c}{I_s \cos \phi_s}$, or $\frac{d^3_c}{d^3_s \cos \phi_s}$ for plain rectangular sections.

If it is assumed that the moment of inertia of the rib varies as the cube of the thickness, Equation (4) can be changed (see Fig. 2) to:

$$d_x = d_c\, c \sqrt[6]{1 + \tan^2 \phi} \quad \text{.....................(5)}$$

in which,

$$c = \frac{1}{\sqrt[3]{1 - (1 - m)\, z}} \quad \text{.....................(5a)}$$

The rib thickness at any point can then be obtained from Equation (5) with the help of Tables 12 and 13 of the writer's previous paper.[8] The value of $\cos \phi_s$ can also be computed from Table 12 of that paper, with the formula,

$$\cos \phi_s = \frac{1}{\sqrt{1 + \tan^2 \phi_s}}$$

When the reinforcement of the rib is such that I does not vary as the cube of d, the value of m in Equation (5) should be computed from d_c and

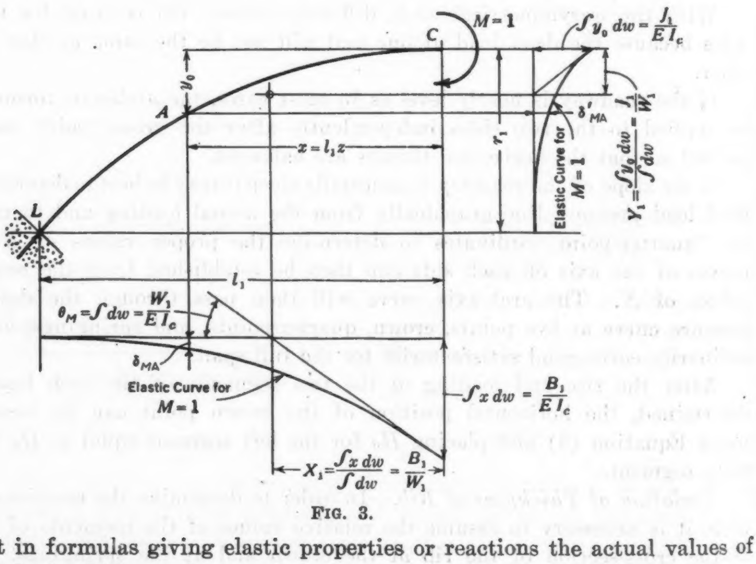

FIG. 3.

d_s, but in formulas giving elastic properties or reactions the actual values of I_c and I_s should be used.

Elastic Properties of Rib Segments.—Consider the arch cut at the crown with the right half removed, leaving the left segment projecting as a canti-

[7] *Loc. cit.*, p. 978.

[8] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), pp. 980-981.

lever from the abutment, as shown in Fig. 3. Formulas will be derived for the elastic deformations of the rib segment produced by unit horizontal and vertical loads and a unit moment applied at the crown point, C, which is the free end of the cantilever.

Using equations for the arch axis and the rib thickness, previously given, the value of $\frac{ds}{E\,I}$ (designated by dw), can readily be derived.[9] The formula is, as follows:

$$dw = \frac{l_1}{E\,I_c}\left[1 - (1 - m)\,z\right] dz \qquad (6)$$

The angular displacement at C produced by a unit moment is:

$$\int M\,dw = \int dw = \frac{l_1}{E\,I_c}\int\left[1 - (1 - m)\,z\right] dz = \frac{l_1}{E\,I_c}\,\frac{1 + m}{2} \qquad (7)$$

or,

$$\int dw = \frac{W_1}{E\,I_c} \qquad (7a)$$

in which, W_1 is a constant for each particular rib segment which can be computed from the values given in Table 1. The vertical displacement at C

TABLE 1.—VALUES OF $\int dw$, $\int x\,dw$, $\int x^2\,dw$, AND X_1 (FOR ONE SEGMENT OF ARCH RIB).

(All intermediate values by interpolation)

Value of m	$\frac{W_1}{l_1}$ Equation (7)	$\frac{B_1}{l^2_1}$ Equation (8)	$\frac{D_1}{l^3_1}$ Equation (13)	$\frac{X_1}{l_1}$ Equation (10)
0.15	0.575	0.2167	0.1208	0.3768
0.20	0.600	0.2333	0.1333	0.3889
0.25	0.625	0.2500	0.1458	0.4000
0.30	0.650	0.2667	0.1583	0.4103
0.35	0.675	0.2833	0.1708	0.4198
0.40	0.700	0.3000	0.1833	0.4286
0.45	0.725	0.3167	0.1958	0.4368
0.50	0.750	0.3333	0.2083	0.4444

produced by the unit moment, is:

$$\int M\,x\,dw = \int x\,dw = \frac{l^2_1}{E\,I_c}\int\left[z - z^2 + m\,z^2\right] dz = \frac{l^2_1}{E\,I_c}\,\frac{1 + 2\,m}{6} \qquad (8)$$

or,

$$\int x\,dw = \frac{B_1}{E\,I_c} \qquad (8a)$$

in which, $B_1 = E\,I_c \int x\,dw$, and can be computed from values in Table 1.

[9] *Loc. cit.*, p. 983.

The horizontal displacement at C produced by the unit moment is, [10]

$$\int M\,y_0\,dw = \int y_0\,dw = \frac{l_1\,r_1}{E\,I_c\,(g-1)}\left[\frac{\sqrt{g^2-1}}{k} - 1 - (1-m)\left(\frac{\sqrt{g^2-1}}{k} - \frac{g-1}{k^2} - \frac{1}{2}\right)\right] \qquad (9)$$

or,

$$\int y_0\,dw = \frac{J_1}{E I_c} \qquad (9a)$$

in which, $J_1 = E\,I_c \int y_0\,dw$, and can be obtained from Table 2(a).

TABLE 2.—Factors for the Solution of Equations (9), (11), (14), and (16). (Intermediate values by interpolation)

N	$m = 0.15$	$m = 0.20$	$m = 0.25$	$m = 0.30$	$m = 0.35$	$m = 0.40$	$m = 0.45$	$m = 0.50$
			(a) Values of $\frac{J_1}{l_1 r_1}$ (Equation (9))					
0.25	0.1208	0.1323	0.1458	0.1583	0.1708	0.1833	0.1958	0.2083
0.23	0.1142	0.1262	0.1383	0.1503	0.1624	0.1744	0.1865	0.1985
0.21	0.1074	0.1190	0.1305	0.1421	0.1537	0.1653	0.1769	0.1885
0.19	0.1006	0.1117	0.1228	0.1339	0.1450	0.1561	0.1673	0.1784
0.17	0.0936	0.1042	0.1149	0.1255	0.1361	0.1468	0.1574	0.1680
0.15	0.0866	0.0968	0.1068	0.1170	0.1271	0.1372	0.1473	0.1575
			(b) Values of $\frac{Y_1}{r_1}$ (Equation (11))					
0.25	0.2101	0.2222	0.2333	0.2436	0.2531	0.2619	0.2701	0.2778
0.23	0.1985	0.2103	0.2212	0.2312	0.2405	0.2491	0.2572	0.2647
0.21	0.1867	0.1983	0.2089	0.2187	0.2277	0.2362	0.2440	0.2513
0.19	0.1748	0.1861	0.1964	0.2060	0.2148	0.2230	0.2307	0.2378
0.17	0.1628	0.1738	0.1838	0.1931	0.2017	0.2097	0.2171	0.2240
0.15	0.1506	0.1612	0.1709	0.1799	0.1883	0.1960	0.2032	0.2099
			(c) Values of $\frac{G_1}{r_1 l^2_1}$ (Equation (14))					
0.25	0.0800	0.0900	0.1000	0.1100	0.1200	0.1300	0.1400	0.1500
0.23	0.0762	0.0859	0.0956	0.1053	0.1150	0.1247	0.1344	0.1441
0.21	0.0725	0.0818	0.0912	0.1006	0.1100	0.1193	0.1287	0.1381
0.19	0.0686	0.0776	0.0867	0.0957	0.1048	0.1138	0.1229	0.1319
0.17	0.0646	0.0733	0.0820	0.0907	0.0994	0.1081	0.1169	0.1256
0.15	0.0606	0.0689	0.0773	0.0856	0.0940	0.1023	0.1107	0.1190
			(d) Values of $\frac{F_1}{r^2_1 l_1}$ (Equation (16))					
0.25	0.0583	0.0666	0.0750	0.0834	0.0916	0.1000	0.1083	0.1167
0.23	0.0533	0.0617	0.0696	0.0775	0.0853	0.0932	0.1012	0.1092
0.21	0.0494	0.0568	0.0643	0.0718	0.0792	0.0866	0.0940	0.1015
0.19	0.0452	0.0522	0.0592	0.0662	0.0732	0.0803	0.0873	0.0943
0.17	0.0410	0.0476	0.0542	0.0608	0.0674	0.0740	0.0806	0.0872
0.15	0.0371	0.0433	0.0495	0.0556	0.0618	0.0680	0.0741	0.0802

Values of g corresponding to N, in Tables 2(a), 2(b), 2(c), and 2(d), are as follows (this applies also to Table 4):

N	g	N	g
0.25	1.000	0.19	4.324
0.24	1.347	0.18	5.321
0.23	1.756	0.17	6.536
0.22	2.240	0.16	8.031
0.21	2.814	0.15	9.889
0.20	3.500		

[10] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 989.

When $g = 1$, the axis is parabolic and Equation (9) becomes:

$$\int y_0 \, dw = \frac{l_1 \, r_1}{E \, I_c} \left(\frac{1}{3} - \frac{1 - m}{4} \right) \ldots\ldots\ldots\ldots\ldots\ldots (9b)$$

The co-ordinates, X_1 and Y_1, of the center of gravity of the values of dw can now be determined, as follows:

$$X_1 = \frac{\int x \, dw}{\int dw} = \frac{B_1}{W_1} = \frac{(1 + 2\,m)\, l_1}{3 + 3\,m} \ldots\ldots\ldots\ldots\ldots (10)$$

and,

$$Y_1 = \frac{\int y_0 \, dw}{\int dw} = \frac{J_1}{W_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The values of X_1 and Y_1 can be computed from Tables 1 and 2(*b*), respectively. The equation of the elastic curve of vertical displacements is derived as follows: The second derivative of the elastic curve equals,[9]

$$\frac{d^2 \, \delta}{d z^2} = - \frac{M \, l^2_1}{E \, I_c} \left[1 - (1 - m)\, z \right]$$

Integrating twice with the proper constants,

$$\delta_{MA} = \frac{l^2_1}{6 \, E \, I_c} \left[-\, (1 - m)\, z^3 + 3 z^2 - (3 + 3m)\, z + (1 + 2m) \right] \ldots (12)$$

The values of $\frac{E \, I_c}{l^2_1} \, \delta_{MA}$ are given in Table 3(*a*).

TABLE 3.—ORDINATES OF ELASTIC CURVE OF ONE SEGMENT, (*a*) WITH UNIT MOMENT, AND (*b*) WITH UNIT VERTICAL FORCE, AT THE CROWN.

(Intermediate values by interpolation)

m	Point 1,* $z = 0.9$	Point 2, $z = 0.8$	Point 3, $z = 0.7$	Point 4, $z = 0.6$	Point 5, (quarter-point); $z = 0.5$	Point 6, $z = 0.4$	Point 7, $z = 0.3$	Point 8, $z = 0.2$	Point 9, $z = 0.1$	Point 10 (crown), $z = 0.0$
(*a*) VALUE OF $\delta_{MA} \frac{E I_c}{l^2_1}$ (EQUATION (12))										
1.00....	0.0050	0.0200	0.0450	0.0800	0.1250	0.1800	0.2450	0.3200	0.4050	0.5000
0.50....	0.0026	0.0107	0.0247	0.0453	0.0729	0.1080	0.1511	0.2026	0.2633	0.3333
0.25....	0.0013	0.0060	0.0146	0.0280	0.0469	0.0720	0.1041	0.1440	0.1924	0.2500
0.15....	0.0009	0.0041	0.0106	0.0211	0.0365	0.0576	0.0853	0.1205	0.1640	0.2167
(*b*) VALUE OF $\delta_{VA} \frac{E I_c}{l^3_1}$ (EQUATION (15))										
1.00....	0.0048	0.0187	0.0405	0.0693	0.1042	0.1440	0.1878	0.2347	0.2835	0.3333
0.50....	0.0025	0.0099	0.0222	0.0389	0.0599	0.0846	0.1125	0.1429	0.1752	0.2083
0.25....	0.0013	0.0056	0.0130	0.0237	0.0378	0.0549	0.0748	0.0971	0.1210	0.1458
0.15....	0.0009	0.0038	0.0093	0.0177	0.0289	0.0430	0.0598	0.0788	0.0993	0.1208

* For $z = 1.0$ (at the springing line, Point 0), $\delta_{MA} \frac{E I_c}{l^2_1}$ and $\delta_{VA} \frac{E I_c}{l^3_1} = 0$ at all values of m.

The displacements produced by a unit vertical load will now be determined. The vertical displacement at C (Fig. 4), is:

$$\int M\,x\,dw = \int x^2\,dw = \frac{l^3_1}{E\,I_c}\int (z^2 - z^3 + m\,z^3)\,dz$$

from which,

$$\int x^2\,dw = \frac{l^3_1}{E\,I_c}\,\frac{1 + 3\,m}{12} = \frac{D_1}{E\,I_c} \quad \ldots\ldots\ldots\ldots\ldots(13)$$

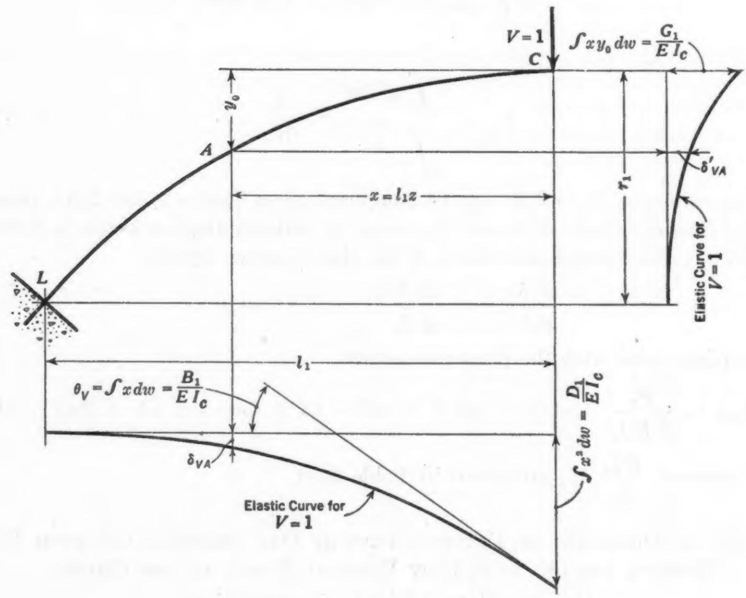

FIG. 4.

in which, $D_1 = E\,I_c \int x^2\,dw$ can be computed from Table 1.

The horizontal displacement at C due to a unit vertical load at C is:

$$\int M\,y_0\,dw = \int x\,y_0\,dw = \frac{r_1\,l^2_1}{E\,I_c\,(g-1)}\left[-\frac{g-1}{k^2} - \frac{1}{2} + \frac{1-m}{3} + \frac{2\,(1-m)}{k^2}\left(g - \frac{\sqrt{g^2-1}}{k}\right) + \frac{m\sqrt{g^2-1}}{k}\right] \ldots\ldots(14)$$

or,

$$\int x\,y_0\,dw = \frac{G_1}{E\,I_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14a)$$

in which, $G_1 = E\,I_c \int x\,y_0\,dw$, and can be obtained from Table 2(c). For

the special case of the parabolic rib when $g = 1$, a simple formula can be derived using the relation, $y_0 = r_1 z^2$. Then,

$$\int x\, y_0\, dw = \frac{r_1 l_1^2}{E I_c} \left(\frac{1}{4} - \frac{1 - m}{5}\right) \ldots\ldots\ldots\ldots\ldots (14b)$$

The equation of the elastic curve of vertical displacements due to the unit vertical load at C is obtained in the same manner as Equation (12) and is:

$$\delta_{VA} = \frac{l_1^3}{12\, E\, I_c} \left[- (1 - m)\, z^4 + 2\, z^3 - (2 + 4\, m)\, z + (1 + 3\, m)\right] \ldots (15)$$

The values of δ_{VA} can be obtained from Table 3(*b*).

The displacements produced by a unit horizontal load will be as follows (see Fig. 5): The vertical displacement of C due to a unit horizontal load

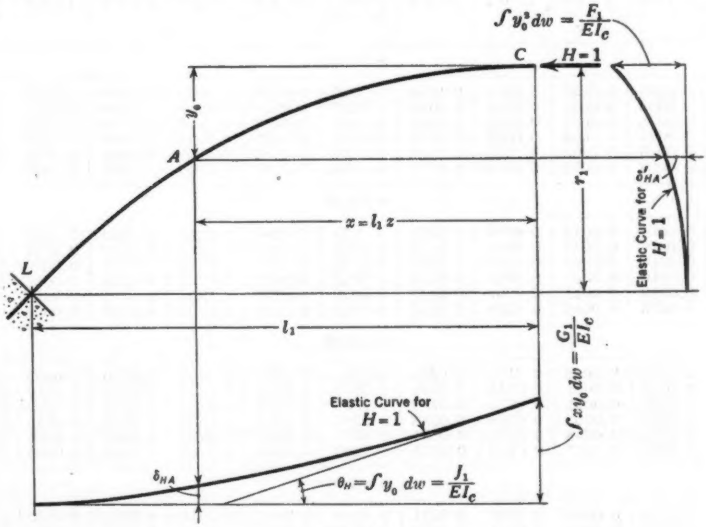

FIG. 5.

at C is equal to the horizontal displacement due to a vertical load and is given by Equations (14) and (14*a*), with Table 2(*c*). The horizontal displacement at C due to a unit horizontal load at C is:

$$\int y_0^2\, dw = Y_1^2 \int dw + \int y^2\, dw$$

in which, y is the ordinate from the axis through the center of gravity of the values of dw.[11]

The horizontal displacement is given by the formula,

$$\int y_0^2\, dw = \frac{F_1}{E\, I_c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The value of F_1, can be taken from Table 2(*d*).

[11] The values of $\int y^2\, dw$ are given in *Transactions*, Am. Soc. C. E., Vol. 88 (1925), Table 18, p 991.

The equation of the elastic curve of vertical displacements due to the unit horizontal load at C is derived from:

$$\frac{d^2\,\delta_{HA}}{dz^2} = \frac{-\,l^2_1\,r_1}{E\,I_c\,(g-1)}\,(\cosh zk - 1)\,[1-(1-m)\,z]$$

The values of δ_{HA} are given in Table 4.

TABLE 4.—Ordinates of Elastic Curve of One Segment with Unit Horizontal Force at Crown; $\left(\text{Values of } \delta_{HA}\,\frac{E\,I_c}{l^2_1\,r_1}\right)$.

(Intermediate values by interpolation)

N*	Point 1,† $z = 0.9$	Point 2, $z = 0.8$	Point 3, $z = 0.7$	Point 4, $z = 0.6$	Point 5 (quarter-point), $z = 0.5$	Point 6, $z = 0.4$	Point 7, $z = 0.3$	Point 8, $z = 0.2$	Point 9, $z = 0.1$	Point 10 (crown) $z = 0.0$
					$m = 0.15$					
0.25....	0.0008	0.0036	0.0083	0.0150	0.0235	0.0333	0.0442	0.0558	0.0679	0.0800
0.23....	0.0008	0.0035	0.0081	0.0146	0.0227	0.0322	0.0426	0.0536	0.0650	0.0764
0.21....	0.0008	0.0035	0.0080	0.0142	0.0220	0.0310	0.0407	0.0512	0.0618	0.0726
0.19....	0.0008	0.0033	0.0078	0.0137	0.0212	0.0297	0.0389	0.0486	0.0586	0.0687
0.17....	0.0008	0.0033	0.0075	0.0132	0.0202	0.0282	0.0369	0.0459	0.0552	0.0646
0.15....	0.0008	0.0032	0.0073	0.0128	0.0194	0.0269	0.0349	0.0433	0.0520	0.0606
					$m = 0.20$					
0.25....	0.0010	0.0043	0.0099	0.0176	0.0273	0.0384	0.0505	0.0634	0.0766	0.0899
0.23....	0.0010	0.0043	0.0097	0.0173	0.0265	0.0371	0.0487	0.0609	0.0734	0.0860
0.21....	0.0010	0.0042	0.0095	0.0168	0.0256	0.0356	0.0466	0.0581	0.0699	0.0819
0.19....	0.0010	0.0041	0.0092	0.0163	0.0247	0.0343	0.0445	0.0554	0.0666	0.0777
0.17....	0.0010	0.0041	0.0091	0.0157	0.0236	0.0327	0.0424	0.0525	0.0629	0.0733
0.15....	0.0010	0.0040	0.0088	0.0151	0.0227	0.0311	0.0403	0.0496	0.0592	0.0689
					$m = 0.25$					
0.25....	0.0013	0.0052	0.0116	0.0204	0.0311	0.0434	0.0569	0.0710	0.0855	0.1000
0.23....	0.0013	0.0051	0.0114	0.0199	0.0302	0.0419	0.0547	0.0682	0.0819	0.0957
0.21....	0.0013	0.0051	0.0111	0.0193	0.0292	0.0404	0.0525	0.0652	0.0782	0.0912
0.19....	0.0013	0.0050	0.0109	0.0188	0.0282	0.0388	0.0503	0.0623	0.0745	0.0867
0.17....	0.0012	0.0048	0.0106	0.0181	0.0271	0.0371	0.0479	0.0591	0.0706	0.0821
0.15....	0.0012	0.0047	0.0103	0.0175	0.0260	0.0355	0.0455	0.0559	0.0666	0.0772
					$m = 0.30$					
0.25....	0.0015	0.0060	0.0133	0.0231	0.0350	0.0485	0.0631	0.0784	0.0942	0.1101
0.23....	0.0015	0.0059	0.0131	0.0225	0.0339	0.0469	0.0608	0.0754	0.0904	0.1054
0.21....	0.0014	0.0058	0.0128	0.0219	0.0329	0.0452	0.0585	0.0724	0.0865	0.1006
0.19....	0.0014	0.0057	0.0124	0.0213	0.0317	0.0434	0.0560	0.0690	0.0823	0.0957
0.17....	0.0014	0.0056	0.0121	0.0206	0.0306	0.0416	0.0534	0.0657	0.0782	0.0907
0.15....	0.0014	0.0055	0.0118	0.0200	0.0294	0.0397	0.0508	0.0623	0.0739	0.0856
					$m = 0.40$					
0.25....	0.0019	0.0076	0.0166	0.0285	0.0426	0.0585	0.0756	0.0936	0.1117	0.1300
0.23....	0.0019	0.0075	0.0164	0.0278	0.0414	0.0566	0.0730	0.0900	0.1073	0.1247
0.21....	0.0019	0.0074	0.0159	0.0270	0.0401	0.0546	0.0701	0.0864	0.1028	0.1193
0.19....	0.0019	0.0073	0.0156	0.0264	0.0388	0.0527	0.0675	0.0829	0.0983	0.1139
0.17....	0.0019	0.0071	0.0152	0.0255	0.0374	0.0505	0.0644	0.0789	0.0935	0.1081
0.15....	0.0019	0.0070	0.0149	0.0247	0.0360	0.0484	0.0615	0.0750	0.0886	0.1023
					$m = 0.50$					
0.25....	0.0024	0.0093	0.0200	0.0339	0.0503	0.0685	0.0881	0.1085	0.1291	0.1500
0.23....	0.0024	0.0091	0.0196	0.0331	0.0489	0.0665	0.0852	0.1046	0.1244	0.1442
0.21....	0.0024	0.0090	0.0191	0.0322	0.0474	0.0641	0.0820	0.1004	0.1192	0.1380
0.19....	0.0023	0.0088	0.0188	0.0314	0.0459	0.0619	0.0788	0.0964	0.1141	0.1320
0.17....	0.0023	0.0087	0.0183	0.0304	0.0443	0.0595	0.0755	0.0920	0.1088	0.1256
0.15....	0.0023	0.0085	0.0178	0.0294	0.0426	0.0570	0.0720	0.0875	0.1033	0.1190

* Corresponding values of g previously listed.

† For $z = 1.0$ (that is, at the springing line, Point 0), $\delta_{HA}\,\frac{E\,I_c}{l^2_1 r_1} = 0$ at all values of N and m.

II.—Reactions of Unsymmetrical Arch

Geometry of Unsymmetrical Rib.—Assume two rib segments of different proportions joined at the crown. In the formulas the principal dimensions of the left segment are indicated with a subscript, 1, and those of the right segment

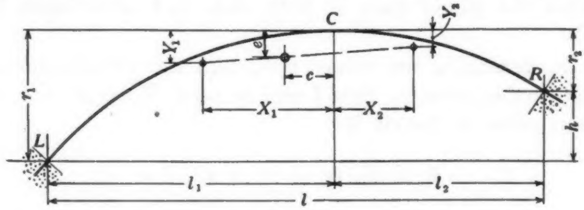

FIG. 6.

with a subscript, 2. Such terms as g, m, d_s, etc., refer to their respective segments without special designation. The crown thickness, d_c, is the same for both segments.

The origin of co-ordinates will be taken at the crown point and the co-ordinates of the center of gravity of the values of dw for the entire rib may be determined by taking moments. Referring to Fig. 6.

$$c = \frac{X_1 W_1 - X_2 W_2}{W_1 + W_2} = \frac{B_1 - B_2}{W_1 + W_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(17)$$

and,

$$e = \frac{Y_1 W_1 + Y_2 W_2}{W_1 + W_2} = \frac{J_1 + J_2}{W_1 + W_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(18)$$

The entire rib will now be considered as a cantilever free at the left end and fixed at the right end, with the reactions, H, V, and M, acting on the left end of the rib through weightless levers. Any reaction that passes through the center of gravity of the values of dw, the co-ordinates of which are c and e, will produce no rotation of the left end, L. By writing equations for the vertical and horizontal displacements of L, the H and V reactions of the fixed arch can be derived.

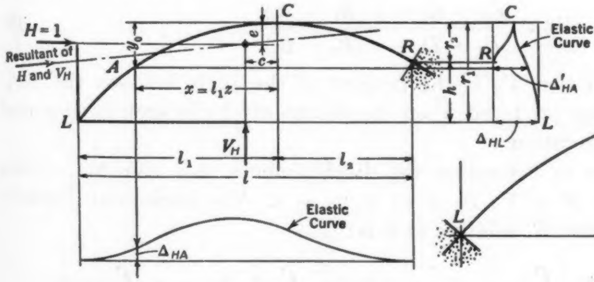

FIG. 7.

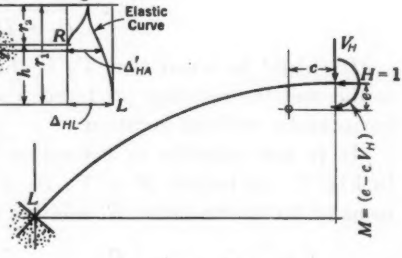

FIG. 8.

Determination of Horizontal Reaction

Assume the arch rib as in Fig. 7 to be acted upon by a unit value of H with a vertical reaction, V_H, of sufficient magnitude to cause the left end, L,

to move in a horizontal direction. In the case of the symmetrical arch, the value of V_H is zero. The elastic curve of vertical displacements of the rib, Δ_{HA}, produced by H and V_H, when divided by the corresponding horizontal displacement, Δ_{HL}, of the left end, L, is the influence line of horizontal reactions when the rib is fixed at both ends and acted upon by a single vertical load.

In order to determine the value of V_H and the deformations of the rib, the elastic properties given in Part I will be used. Since $H = 1$, the bending moment at any point in the rib is:

$$M = V_H\,(x + c) + y_0 - e$$

assuming, c to be positive, and x to be positive to the right and negative to the left of the crown point. The vertical displacement of the crown point, C, in relation to L, due to the bending in the left segment will then be:

$$\int_c^L M\,x\,dw = V_H\left(\int x^2\,dw + c\int x\,dw\right) + \int x\,y_0\,dw - e\int x\,dw$$

or,

$$E\,I_c\int_c^L M\,x\,dw = V_H\,(-D_1 + c\,B_1) + G_1 - e\,B_1$$

for the right segment the displacement of C relative to R is:

$$E\,I_c\int_R^c M\,x\,dw = V_H\,(D_2 + c\,B_2) + G_2 - e\,B_2$$

Since the vertical displacement of L relative to R is zero, it follows that:

$$E\,I_c\int_R^L M\,x\,dw = V_H\,(-\,D_1 - D_2 + cB_1 - cB_2) + G_1 - G_2 - eB_1 + eB_2 = 0$$

from which,

$$V_H = \frac{G_1 - G_2 - e\,(B_1 - B_2)}{D_1 + D_2 - c\,(B_1 - B_2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

It should be noted that V_H is the tangent of the angle between the horizontal and the reaction produced when the abutment of the arch is displaced horizontally without rotation.

It is now possible to determine the displacements, Δ_{HL} and Δ_{HA}, shown in Fig. 7. As before, $M = V_H\,(x + c) + y_0 - e$. The horizontal displacement of the crown point, C, relative to L is:

$$\int M\,y_0\,dw = V_H\int (x\,y_0 + c\,y_0)\,dw + \int y_0^2\,dw - e\int y_0\,dw$$

or,

$$E\,I_c\int M\,y_0\,dw = -\,V_H\,(G_1 - c\,J_1) + F_1 - e\,J_1$$

For the right segment the horizontal displacement of C relative to R is:

$$E\, I_c \int M\, y_0\, dw = V_H\, (G_2 + c\, J_2) + F_2 - e\, J_2$$

The total displacement is then equal to the sum of the displacements for the two sides, hence,

$$\Delta_{HL}\, E\, I_c = F_1 + F_2 - e\,(J_1 + J_2) - V_H\,[(G_1 - G_2) - c\,(J_1 + J_2)] \quad \text{..(20)}$$

Since $e\,(B_1 - B_2) = c\,(J_1 + J_2)$, Equation (20) can be written:

$$\Delta_{HL}\, E\, I_c = F_1 + F_2 - e\,(J_1 + J_2) - V_H\,[G_1 - G_2 - e\,(B_1 - B_2)] \quad \text{..(20a)}$$

In order to obtain the elastic curve of the rib produced by $H = 1$, consider the two segments again separated as in Part I. The forces acting on the left-hand segment are then shown in Fig. 8. The vertical displacement of any point, A, is then equal to:

For the left-hand segment:

$$\Delta_{HA} = \delta_{HA} - V_H\, \delta_{VA} - (e - c\, V_H)\, \delta_{MA} \quad \text{............(21L)}$$

For the right-hand segment:

$$\Delta_{HA} = \delta_{HA} + V_H\, \delta_{VA} - (e - c\, V_H)\, \delta_{MA} \quad \text{............(21R)}$$

and the horizontal reaction due to a vertical load, P, at the point, A, is:

$$H = P\, \frac{\Delta_{HA}}{\Delta_{HL}} \quad \text{........................(22)}$$

All the values in these formulas can be obtained from the tables. The values of δ are, of course, not the same for the two segments.

Determination of Vertical Reaction

Assume a unit vertical force, $V = 1$, acting on the left end of the arch rib and passing through the center of gravity of the values of dw as shown in Fig. 9. In order to make the left end move vertically it will be necessary to

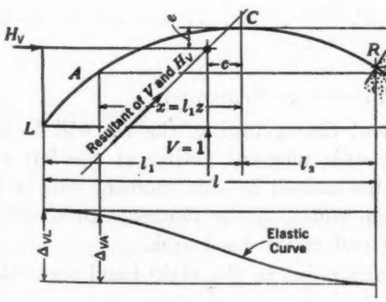

FIG. 9.

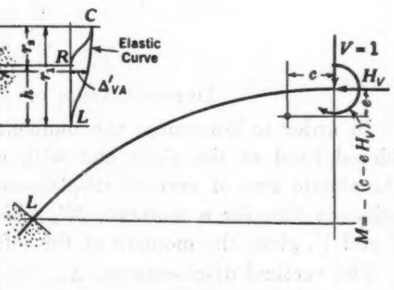

FIG. 10.

apply a horizontal force, H_V, also acting through the center of gravity, so that there will be no rotation at L. If the arch were symmetrical, H_V would be zero.

The magnitude of H_V can be obtained by writing the equation for the horizontal displacement of L and making it equal to zero. This can be done as before in determining V_H, thus:

$$M = (x + c) + H_V (y_0 - e)$$

Following the procedure used in the derivation of Equation (20) the horizontal displacement of L will be:

$$E I_c \int M y_0 \, dw = H_V [F_1 + F_2 - e (J_1 + J_2)] - (G_1 - G_2) + c (J_1 + J_2)$$

or,

$$H_V = \frac{G_1 - G_2 - c (J_1 + J_2)}{F_1 + F_2 - e (J_1 + J_2)} = \frac{G_1 - G_2 - e (B_1 - B_2)}{F_1 + F_2 - e (J_1 + J_2)} \; \ldots\ldots \; (23)$$

The value of H_V is also the tangent of the angle between the vertical and the resultant when the abutment is displaced vertically without rotation.

The influence line for vertical reaction can now be found in the same manner as the horizontal reaction. The vertical displacement of the left end produced by $V = 1$ will be, from the value of $\int M \, x \, dw$ used in the derivation of Equation (19) with the proper values of V and H_V substituted:

$$\Delta_{VL} \, E \, I_c = D_1 + D_2 - c (B_1 - B_2) - H_V [G_1 - G_2 - e (B_1 - B_2)] \ldots (24)$$

The elastic curve of vertical displacements is obtained as for H. For the left segment the vertical displacement at any point, A (Fig. 10), is:

$$\Delta_{VA} = \Delta_{VL} - \delta_{VA} + H_V \, \delta_{HA} + (c - e \, H_V) \, \delta_{MA} \; \ldots\ldots\ldots (25L)$$

and for the right segment:

$$\Delta_{VA} = \delta_{VA} + H_V \, \delta_{HA} + (c - e \, H_V) \, \delta_{MA} \; \ldots\ldots\ldots\ldots (25R)$$

The vertical reaction due to a vertical load, P, at A is, then:

$$V = P \, \frac{\Delta_{VA}}{\Delta_{VL}} \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Determination of Moment at Springing

In order to determine the moment at the springing, the rib will be considered fixed at the right end with a unit moment acting at the left end. The elastic line of vertical displacements caused by this moment will be the influence line for a moment, M', which, added to the moments produced by H and V, gives the moment at the left end of the fixed arch.

The vertical displacement, Δ_{MA}, at any point in the right-hand segment is given by Equation (12):

$$\Delta_{MA} \, E I_c = \delta_{MA} \, E I_c = \frac{l^2_2}{6} [(1 - m) z^3 - 3 z^2 + (3 + 3 m) z - (1 + 2m)] \ldots (27R)$$

Values of $\Delta_{MA} \, E \, I_c$ are given in Table 3(a).

For the left segment the ordinate, Δ_{MA}, of the elastic curve is made up of two parts, a and b. From Fig. 11 it will be seen that,

$$\theta_{ML} = \int dw = \frac{1}{E\,I_c}\,(W_1 + W_2)$$

and,

$$b = \theta_{ML}\,x - \frac{B_1 - B_2}{E\,I_c} = \frac{1}{E\,I_c}\,[l_1\,(W_1 + W_2)\,z - (B_1 - B_2)]$$

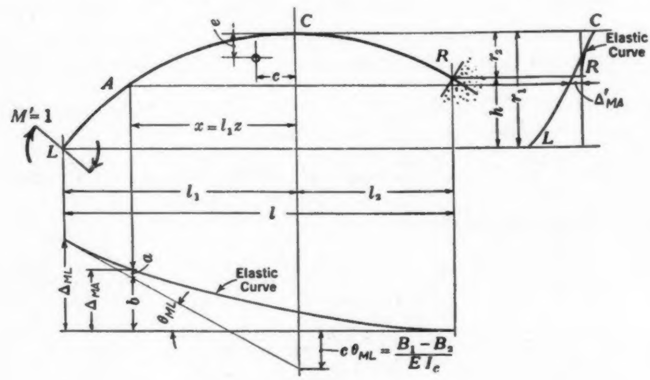

FIG. 11.

The value of a is given by Equation (12); thus:

$$a\,E\,I_c = \delta_{MA}\,E\,I_c = \frac{l_1^2}{6}\,[(1 - m)z^3 - 3z^2 + (3 + 3m)z - (1 + 2m)]$$

Then,

$$\Delta_{MA}\,E\,I_c = (a + b)\,E\,I_c = l_1\,(W_1 + W_2)z - (B_1 - B_2)$$
$$+ \frac{l_1^2}{6}\,[(1 - m)z^3 - 3z^2 + (3 + 3m)z - (1 + 2m)] \ldots\ldots (27L)$$

The vertical displacement at the left end will be equal to,

$$\Delta_{ML} = (l_1 - c)\theta_{ML} = (l_1 - c)\;(W_1 + W_2)\;\frac{1}{E\,I_c}$$

the value of M' is,

$$M' = \frac{\Delta_{MA}}{\theta_{ML}} = \frac{\Delta_{MA}\,E\,I_c}{W_1 + W_2} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The moment at the left springing of the fixed arch produced by a concentrated vertical load is:

$$M = M' + H\,(r_1 - e) - V\,(l_1 - c) \ldots\ldots\ldots\ldots\ldots (29)$$

EFFECT OF HORIZONTAL LOADS

The effect of horizontal loads has not been treated in detail because in practical design it is seldom necessary to consider it. However, if it is

desired to do so, the reactions produced by a horizontal load at any point can be determined by substituting the values of Δ'_{HA}, Δ'_{VA}, and Δ'_{MA}, for Δ_{AH}, Δ_{VA}, and Δ_{MA}, respectively, in Equations (22), (26), and (28) (see Figs. 7, 9, and 11).

Effect of Temperature Change

If the left end of the arch rib were free, the effect of a change in temperature would be to lengthen or to shorten the chord of the arch, by an amount equal to $t\,\alpha\,\sqrt{l^2 + h^2}$, producing a corresponding displacement of the left end. Since the left end is actually fixed, the reactions produced by the temperature change will be those required to return the end to its original position.

Considering the horizontal component of the temperature displacement, $t\,\alpha\,l$, only: $H = \dfrac{t\,\alpha\,l}{\Delta_{HL}}$, and $V = V_H \dfrac{t\,\alpha\,l}{\Delta_{HL}}$. Considering the vertical component, $t\,\alpha\,h$, only: $V = \dfrac{t\,\alpha\,h}{\Delta_{VL}}$, and $H = H_V \dfrac{t\,\alpha\,h}{\Delta_{VL}}$. Then, the reactions at L, due to temperature change, are:

$$H_t = \frac{t\,\alpha\,l}{\Delta_{HL}} + H_V \frac{t\,\alpha\,h}{\Delta_{VL}} \quad\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

$$V_t = \frac{t\,\alpha\,h}{\Delta_{VL}} + V_H \frac{t\,\alpha\,l}{\Delta_{HL}} \quad\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

and,

$$M_t = H_t\,(r_1 - e) - V_t\,(l_1 - c) \ldots\ldots\ldots\ldots\ldots (32)$$

Effect of Rib-Shortening or Abutment Yielding

Let Δx represent the horizontal component of rib-shortening due to full load on the arch. Then,

$$\Delta x = -H \int_L^R \frac{dx}{\cos\phi\,E\,A} = -H\left(\frac{l_1}{E\,A'_m} + \frac{l_2}{E\,A'_m}\right) \ldots\ldots (33)$$

in which, $A'_m = \dfrac{l_1}{\displaystyle\int_L^c \frac{dx}{A\cos\phi}}$, or $\dfrac{l_2}{\displaystyle\int_R^c \frac{dx}{A\cos\phi}} = A_c\,C'_m$; A_c is the area of cross-section of the rib at the crown; and C'_m is a coefficient given by Fig. 12. The value of A'_m must be computed from each segment of rib separately. The vertical component of rib-shortening is:

$$\Delta y = -H \int_L^R \frac{dy}{\cos\phi\,E\,A} = -H\left(\frac{r_1}{E\,A''_m} - \frac{r_2}{E\,A''_m}\right) \ldots\ldots (34)$$

in which, $A''_m = \dfrac{r_1}{\displaystyle\int_L^c \frac{dy}{A\cos\phi}}$, or $\dfrac{r_2}{\displaystyle\int_R^c \frac{dy}{A\cos\phi}} = A_c\,C''_m$; C''_m is given by Fig. 12 and, as before, A''_m must be computed for each segment of rib. These values of Δx and Δy can be substituted in Equations (30) and (31) for

the values of $t \alpha l$ and $t \alpha h$, respectively, and the reactions at L due to rib-shortening are:

$$H_{RS} = \frac{\Delta x}{\Delta_{HL}} + H_V \frac{\Delta y}{\Delta_{VL}} \quad \text{(35)}$$

$$V_{RS} = \frac{\Delta y}{\Delta_{VL}} + V_H \frac{\Delta x}{\Delta_{HL}} \quad \text{(36)}$$

and,

$$M_{RS} = H_{RS} (r_1 - e) - V_{RS} (l_1 - c) \quad \text{(37)}$$

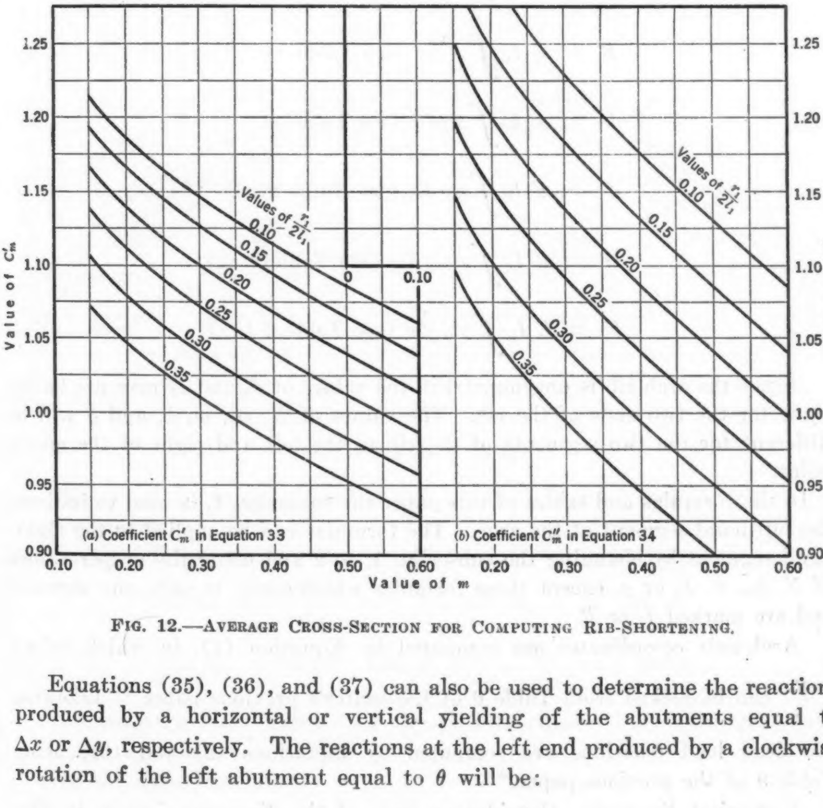

FIG. 12.—AVERAGE CROSS-SECTION FOR COMPUTING RIB-SHORTENING.

Equations (35), (36), and (37) can also be used to determine the reactions produced by a horizontal or vertical yielding of the abutments equal to Δx or Δy, respectively. The reactions at the left end produced by a clockwise rotation of the left abutment equal to θ will be:

$$M' = \frac{\theta E I_c}{W_1 + W_2} \quad \text{(38)}$$

$$H_L = \frac{\theta (r_1 - e)}{\Delta_{HL}} - H_V \frac{\theta (l_1 - c)}{\Delta_{VL}} \quad \text{(39)}$$

$$V_L = -\frac{\theta\,(l_1 - c)}{\Delta_{VL}} + V_H\,\frac{\theta\,(r_1 - e)}{\Delta_{HL}} \qquad (40)$$

and,

$$M_L = M' + H_L\,(r_1 - e) - V_L\,(l_1 - c) \qquad (41)$$

III.—Example

Tables and Formulas.—The integrals of functions of the elastic weight, dw, for each segment of the arch rib, are designated as follows:

$$W = E\,I_c \int dw \text{ (see Table 1)}$$

$$B = E\,I_c \int x\,dw \text{ (see Table 1)}$$

$$D = E\,I_c \int x^2\,dw \text{ (see Table 1)}$$

$$J = E\,I_c \int y_0\,dw \text{ (see Table 2}(a))$$

$$G = E\,I_c \int x\,y_0\,dw \text{ (see Table 2}(c))$$

$$F = E\,I_c \int y_0^2\,dw \text{ (see Table 2 }(d))$$

Since the arch rib is unsymmetrical the values of d_s and I_s may not be the same for the two ends of the rib. The values of N, ϕ_s, m, k, and g will be different for the two segments of the rib to the left and right of the crown point.

In the formulas and tables of this paper the subscript, 1, is used to indicate the left-hand segment of the arch. The formulas can be applied to the right-hand segment by changing the subscript, 1, to 2 and using the proper values of N, ϕ_s, m, k, or g, except those formulas which apply to only one segment and are marked L or R.

Arch-axis co-ordinates are computed by Equation (1), in which values of $\frac{y_0}{r}$ can be picked from Table 6 of the writer's previous paper.[12] Likewise, the dead load reactions are computed by Equations (2) and (3), using Table 9 of the previous paper.[13]

Numerical Example.—Consider an arch of the dimensions given in Fig. 13 with the following dead loads per linear foot: w_c = 520 lb; w_{s1} = 1 030 lb; w_{s2} = 1 000 lb. Then, $g_1 = \frac{w_{s1}}{w_c} = 1.981$; $g_2 = \frac{w_{s2}}{w_c} = 1.923$.

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[12] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 973.
[13] *Loc. cit.*, p. 976.

The crown point is located in the following manner: Taking the values of C_d from Table 9 of the previous paper.[13]

$$H_{d1} = \frac{4\, C_{d1}\, w_c\, l^2_1}{r_1} = H_{d2} = \frac{4\, C_{d2}\, w_c\, l^2_2}{r_2}$$

$$l_1 = l_2 \sqrt{\frac{r_1}{r_2}\left(\frac{C_{d2}}{C_{d1}}\right)} = 1.1503\ l_2$$

Since $l_1 + l_2 = 140$ ft; and, $l_2 = \dfrac{140}{2.1503} = 65.11$ ft; $l_1 = 140 - 65.11 = 74.89$ ft; $H_{d1} = 41\ 938$ lb; and $H_{d2} = 41\ 943$ lb.

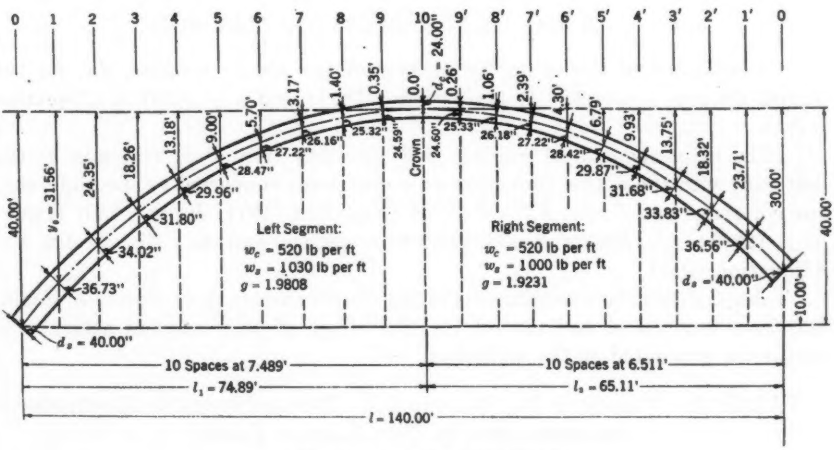

FIG. 13.—ARCH DIMENSIONS

The values of $\tan \phi_s$, m, and N may now be computed (see Table 9 of the previous paper.[13])

For the left segment:

$$\tan \phi_{s1} = \frac{4.553 \times r_1}{2\, l_1} = 1.21592;\ \phi_{s1} = 50^\circ\ 33'\ 50''$$

$$\cos \phi_{s1} = 0.63522;\text{ and } m_1 = \frac{d^3_c}{d^3_s \cos \phi_{s1}} = 0.34004$$

Interpolating from Table 6 of the previous paper[12]: $N_1 = 0.22515$; and y_0 (Point 5) $= N_1\, r_1 = 9.006$ ft.

For the right segment:

$$\tan \phi_{s2} = 1.04224;\ \phi_{s2} = 46^\circ\ 11'\ 04'';\ m_2 = 0.31198$$

$$N_2 = 0.2264;\text{ and } y_0 \text{ (Point 5)} = 6.792 \text{ ft.}$$

The rib-thickness values shown in Fig. 13 were computed with Equations (5) and (5*a*), using Tables 12 and 13 of the previous paper.[14]

[14] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 980.

The following values are computed by taking the values from the tables as noted and multiplying by the corresponding values of l and r:

$W_1 = 50.177$; $W_2 = 42.713$ from Table 1

$B = 1\ 570.0$; $B_2 = 1\ 148$ from Table 1

$J_1 = 473.3$; $J_2 = 296.51$ from Table 2(*a*)

$X_1 = 31.30$ ft; $X_2 = 26.87$ ft. from Table 1

$Y_1 = 9.43$ ft; $Y_2 = 6.94$ ft. from Table 2(*b*)

$D_1 = 70\ 690$; $D_2 = 44\ 520$ from Table 1

$G_1 = 25\ 100$; $G_2 = 13\ 600$ from Table 2(*c*)

$F_1 = 9\ 874$; $F_2 = 4\ 588$ from Table 2(*d*)

Co-ordinates of the center of gravity of the elastic weights, dw, for the entire rib are: $c = 4.524$ ft (Equation (178)); and $e = 8.287$ ft (Equation (18)).

The displacements of the left end resulting from unit reactions at the left end, with the entire rib acting as a cantilever supported at the right end, are computed as follows: $V_H = 0.07064$ (Equation (19)); $H_V = 0.9901$ (Equation (23)); $\Delta_{HL}\ EI_c = 7\ 438$ (Equation (20)); and $\Delta_{VL}\ EI_c = 105\ 376$ (Equation (24)).

Table 5 shows the computations of the displacements, δ, of Points A resulting from unit forces and moments at the crown of each segment acting as a cantilever supported at the springing.

TABLE 5.—VALUES OF δ_{HA}, δ_{VA}, AND δ_{MA}. DISPLACEMENTS OF CANTILEVER SEGMENTS DUE TO UNIT LOAD AT CROWN

Point*	x	$\frac{\delta_{HA}\ EI_c}{l^2 r_1}$ (From Table 4)	$\delta_{HA}\ EI_c$	$\frac{\delta_{VA}\ EI_c}{l^3_1}$ (From Table 3 (*b*))	$\delta_{VA}\ EI_c$	$\frac{\delta_{MA}\ EI_c}{l^2_1}$ (From Table 3 (*a*))	$\delta_{MA}\ EI_c$
(*a*) LEFT SEGMENT, WITH $l_1 = 74.89$ FEET; $r_1 = 40.00$ FEET; $g = 1.981$; $N = 0.225$; AND $m = 0.340$.							
2	0.8	0.0065	1 458	0.0071	2 982	0.0077	43.2
4	0.6	0.0245	5 496	0.0292	12 265	0.0342	191.8
6	0.4	0.0503	11 284	0.0656	27 553	0.0850	476.7
8	0.2	0.0804	18 037	0.1136	47 714	0.1651	926.0
10	0.0	0.1119	25 104	0.1683	70 690	0.2800	1 570.4
(*b*) RIGHT SEGMENT, WITH $l_2 = 65.11$ FEET; $r_2 = 30.00$ FEET; $g = 1.923$; $N = 0.226$; AND $m = 0.312$.							
2	0.8	0.0061	776	0.0066	1 822	0.0071	30.1
4	0.6	0.0230	2 925	0.0275	7 591	0.0323	136.9
6	0.4	0.0477	6 066	0.0622	17 169	0.0809	343.0
8	0.2	0.0765	9 729	0.1084	29 921	0.1585	671.9
10	0.0	0.1069	13 595	0.1613	44 522	0.2707	1 147.6

* For Points 0 all values of $\delta = 0$

Table 6 shows the method of computing: (1) The total displacements, Δ, of Points A, resulting from the unit reactions at the left end with the entire rib acting as a cantilever supported at the right end; (2) the reactions at

the left end resulting from unit loads at Points A with the rib fixed at both ends. The reactions at any other point may then be computed by statics.

TABLE 6.—VALUES OF REACTIONS; H, V, AND M, AT LEFT END OF ARCH, DUE TO UNIT LOADS

Point*	z	$\Delta_{HA} E I_c$ Equation (21)	$H = \frac{\Delta_{HA}}{\Delta_{HL}}$	$\Delta_{VA} E I_c$ Equation (25)	$V = \frac{\Delta_{VA}}{\Delta_{VL}}$	$\Delta_{MA} E I_c$ Equation (27)	M' Equation (28)	M_L Equation (29)
(a) LEFT SEGMENT.								
2	0.8	903	0.1214	103 679	0.9839	5 186	55.83	−9.55
4	0.6	3 102	0.4170	97 847	0.9286	3 943	42.45	−9.67
6	0.4	5 540	0.7448	87 240	0.8279	2 838	30.55	−4.09
8	0.2	7 289	0.9800	72 111	0.6843	1 896	20.41	+3.34
10	0.0	7 600	1.0216	53 760	0.5102	1 148	12.36	+8.86
(b) RIGHT SEGMENT								
2	0.8	665	0.0894	2 479	0.0235	30.1	0.32	+1.51
4	0.6	2 369	0.3185	9 983	0.0947	136.9	1.47	+4.91
6	0.4	4 546	0.6112	21 912	0.2079	343.0	3.69	+8.44
8	0.2	6 490	0.8725	36 981	0.3510	671.9	7.23	+10.20
10	0.0	7 600	1.0216	53 758	0.5102	1147.6	12.36	+9.86

* For Points 0, reactions = 0 except $V = 1$.

The reactions due to temperature change are computed, as follows:

$$H_t \frac{E I_c}{t\,\alpha} = \frac{l}{\Delta_{HL}\, E I_c} + \frac{H_V\, l}{\Delta_{VL}\, E I_c} = 0.018916 \text{ (Equation (30))}$$

$$V_t \frac{E I_c}{t\,\alpha} = \frac{h}{\Delta_{VL}\, E I_c} + \frac{V_H\, l}{\Delta_{HL}\, E I_c} = 0.001425 \text{ (Equation (31))}$$

$$M_t \frac{E I_c}{t\,\alpha} = H_t \frac{E I_c}{t\,\alpha}\,(r_1 - e) - V_t \frac{E I_c}{t\,\alpha}\,(l_1 - c) = 0.4995 \text{ (Equation (32))}$$

In similar manner the reactions due to rib-shortening may be computed with Equations (33) to (37), inclusive, taking values of C'_m and C''_m from Fig. 12.

DISCUSSION OF UNSYMMETRICAL ARCH DESIGN

The method given in this paper establishes the influence lines for moments and thrusts in an unsymmetrical arch when the crown or thinnest section is at the top of the arch where the tangent to the rib axis is horizontal. The thickness need not be the same at the two springings. If one springing is raised much above the other, the unsymmetrical arch will probably be more satisfactory with the thinnest section at the top than at the center of the span. For both appearance and strength it will then ordinarily be better to make the rib thicker at the lower springing than at the upper.

If the high end is too near the top (that is, if $\frac{h}{r_1}$ is too great), it may be necessary to move the thinnest section nearer the center of the span, in order to avoid excessive moments at the crown and at the high end or an excessive rate of increase in rib thickness from the crown to the high end. Although

the variation of the special conditions which control different arches is too great to be able to generalize accurately, the computations for maximum stresses in ribs of various proportions (omitted herein for the sake of brevity) indicate that even when $\frac{h}{r_1}$ is as high as $\frac{3}{4}$, the crown may be satisfactorily located at the top. Although, in this case, the live load bending is greater at the crown than at the center, the temperature, rib-shortening, and shrinkage moments are less at the crown. The actual relation between the maximum moments at the crown and center points in any particular case depends on the relative importance of the live load moments and the moments induced by temperature change, etc.

Assuming that the proper crown thickness has been established, a suitable thickness for the rib at the springing lines can be computed ordinarily by placing m equal to about 0.3 in the formula, $m = \frac{d^3_c}{d^3_s \cos \phi_s}$.

Since the $\cos \phi_s$ is greater at the high end than at the low end, using the same value of m for both ends will result in a greater value of d_s at the low end than at the high end. In an arch similar to that described in the "Numerical Example," but with $\frac{h}{r_1} = \frac{3}{4}$, this method gave values of d_s equal to 40 in. and 37 in., and the calculated maximum stresses were found to be very nearly equal at the two ends. Many designers have a tendency to make the springings too thick in relation to the crown thickness.

The relations between a symmetrical arch and certain type of unsymmetrical arch, shown by Carl B. Andrews,[15] Assoc. M. Am. Soc. C. E., are valuable in making a preliminary design for an unsymmetrical arch and for the final design of an arch which is so nearly symmetrical that the assumed form is satisfactory.

[15] "Reactions for a Particular Type of Unsymmetrical Arch," by Carl B. Andrews, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 438.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

DEFORMATION OF STEEL REINFORCEMENT DURING AND AFTER CONSTRUCTION

BY SERGIUS I. SERGEV,[1] ESQ.

SYNOPSIS

The object of this paper is to compare, during and after construction, observed unit stresses in longitudinal steel reinforcement in helically reinforced concrete columns and **T**-beams with calculated theoretical unit stresses and with allowed unit stresses.

Three columns of high strength concrete, and three columns and six **T**-beams of average strength concrete were under observation. The calculated theoretical unit stresses due to loads were obtained, neglecting the effect of the helical reinforcement and employing ratios of the modulus of elasticity of the steel to the modulus of elasticity of the concrete ordinarily used in designs. No claim is made for the absolute accuracy of these stresses.

The allowed unit stresses as recommended by the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete[2] are empirical. A comparison of these stresses with the observed stresses indicates the departure of the actual from the anticipated. Such a comparison also shows the validity of using allowed unit stress as an indicator of the actual unit stress prevailing in the member.

This paper is presented not as a completed study on the subject of shrinkage, but as a forerunner for future investigations.

INTRODUCTION

Strength designs for reinforced concrete, like those for other materials of construction, are based upon supposedly rational assumptions from which theoretical equations have been deduced. The standard design equations for concrete structures do not account for stresses in the steel and the concrete

NOTE.—Discussion on this paper will be closed in **January, 1934,** *Proceedings.*

[1] Asst. Prof. of Civ. Eng., Univ. of Washington, Seattle, Wash.

[2] *Proceedings,* Am. Soc. C. E., October, 1924, Papers and Discussions, p. 1209.

induced by the shrinkage of the latter, although experience and observations indicate that this neglected factor is of prime importance.

Concrete shrinks as it dries. This action induces initial stresses, compressive in the steel and tensile in the concrete. Experimental evidence indicates that these induced stresses, the magnitudes of which are not generally recognized, are very high.

Shrinkage effects have been studied on a laboratory scale. Some readings on actual buildings have been taken, but to the writer's knowledge when this problem was begun, no investigations had been undertaken to measure the deformations of reinforcement in ordinary structural members in the early stages of construction, A review of the findings on the subject of shrinkage in concrete structures shows that a study of the effect of shrinkage on the deformation of steel during and after construction is a needed contribution to engineering knowledge.

Results of measurements—during and after construction—of hard-grade steel used in the building of Biology Hall at the University of Washington, Seattle, Wash., is presented herein. This is a five-story, reinforced concrete building of a beam-and-slab type. Observations were made on two faces of three columns of 1:1:2 concrete, three columns of $1:2\frac{1}{4}:3\frac{3}{4}$ concrete, and on each face of six beams of $1:2\frac{1}{4}:3\frac{3}{4}$ concrete. Readings were taken for a period of about two years, beginning on February 11, 1930.

The purpose of this work may be summarized as follows: (1) To observe the effect of shrinkage and flow of the concrete on the deformation of steel reinforcement; (2) to note whether the concrete and the steel act together during the early period of setting of the concrete; (3) to note the actual stresses in the reinforcement; (4) to observe the magnitude of the initial stresses; (5) to compare the observed stresses with the calculated theoretical stresses due to actual loads, and with the allowed stresses; and (6) to note the variation in the deformations over a long period of time. Fig. 1 shows the typical construction details in the building.

Instruments Used and Discussion of Readings

An 8-in. Berry strain-gauge with long legs was used, giving readings to $\frac{1}{40\,000}$ in. directly, and closer by interpolation. The temperature of the surrounding atmosphere was taken, but not of the specimens themselves. This introduces an error when the temperature difference between any two readings is large and the time interval small. The temperature of the specimen might be considered as lagging behind the readings. It was noted that different temperatures gave different zero readings on the reference temperature bar provided with the instrument; for example, for a temperature of 40°F the bar reading was about 221, whereas for a temperature of 80°F the bar reading was about 228. This condition is explained by noting that the materials used for the strain-gauge frame and the reference temperature bar have different coefficients of expansion. A comparison of the coefficients of expansion of the reinforcement steel and of the reference temperature bar,

showed the two to be practically equal. Thus, by taking the bar readings with the actual readings of the specimen, the deformation due to temperature

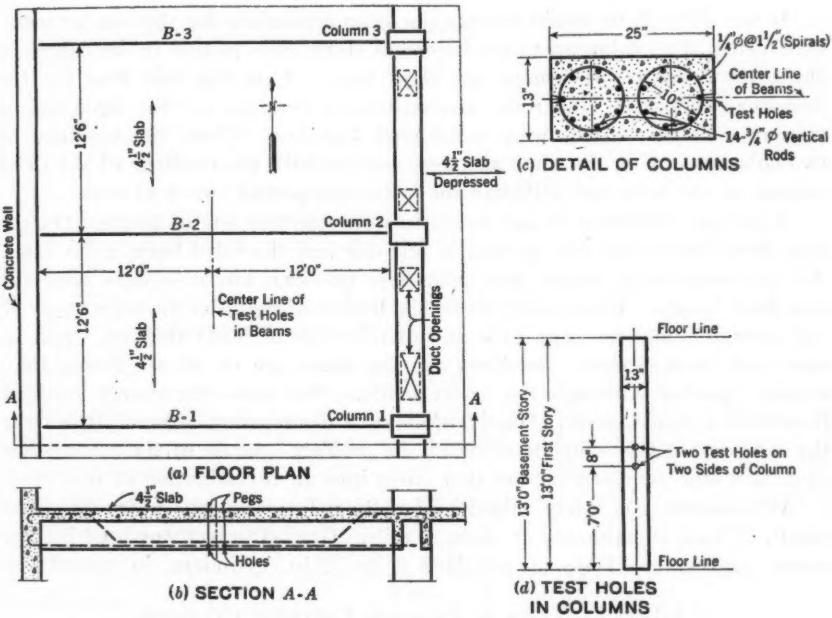

FIG. 1.—TYPICAL DETAILS OF BUILDING TESTED.

changes could be accounted for; for example, the deformation in a bar of one of the columns is obtained as follows:

	February 11, 1930 (no load; temperature 45° F)	February 13, 1930 (temperature, 40° F)
Actual reading of specimen..........	262.5	256.5
Reference bar reading..............	222.0	221.0
Difference	40.5	35.5

The shortening in this case, in 8 in., is found to be $\dfrac{40.5 - 35.5}{5\,000} = 0.001$ in.

PROCEDURE OF TEST

Steel test specimens were taken at random from a stock-pile. Holes were drilled at the proper places and initial readings taken before the specimens were placed in the forms. The next readings were taken as soon as possible after the concrete had been poured, generally from one to two days later. Readings were then taken daily for the first seven days, weekly for the first month, and monthly for the first six months. Additional readings were taken when shoring was removed. In a few cases, because of inaccessibility, the readings were not recorded. Every deformation reading is the average of

two values, the instrument being turned end for end to eliminate errors. No effort was made to regulate the construction to suit the needs peculiar to the test.

It was difficult to obtain exactly the same conditions for the similar members tested. Two columns, in the basement story, were poured in one day while the third column was poured two days later. This was also true for the first-story columns. Under the circumstances, readings for the same age of similar members tested, were considered together. Thus, the readings of two columns having the same age were studied with the readings of the third column of the same age, although the latter was poured two days later.

A greater difference in age exists between the two sets of beams. On the first floor, two beams were poured in one day and the third beam a day later. All the second-floor beams were poured in one day, but nine days after the first-floor beams. Practically, these six beams are similar in percentage of end restraint, and in magnitude and distribution of load; they are equal in span and cross-section. Readings for the same age of all six beams were studied together, although the writer realizes that some discrepancy resulted from such a combination. On the whole, this discrepancy detracts little from the accuracy of the complete study of the problem because of the many other uncertain and unknown factors that enter into an investigation of this kind.

All concrete was ready mixed and delivered to the job in trucks. The length of haul is estimated at about ½ mile. Graded gravel was used for the coarse aggregate. Tests of standard 6 by 12-in. cylinders, in accordance

TABLE 1.—TESTS OF STANDARD CONCRETE CYLINDERS.

Location	Mix	Ultimate stress, in pounds per square inch	Number of cylinders tested
Basement columns	1:1:2	5 000	3
First-story columns	1:2¼:3¾	3 000	3
First-floor beams and slabs	1:2¼:3¾	3 750	4
Second-floor beams and slabs	1:2¼:3¾	3 750	4

with the Building Code of the City of Seattle, showed the ultimate unit stresses at the age of 28 days to be as indicated in Table 1. No concrete was poured during freezing or near-freezing weather.

NOTATION

The following symbols, adopted in this paper, are presented for the guidance of discussers:

a = actually computed.

c = concrete.

e = estimated.

f = unit stress, in pounds per square inch; subscripts: s = steel; c = concrete; u = ultimate strength; t = theoretical; r = shrinkage; a = actually computed; o = observed; superiors: f' = tensile stress; f'' = compressive stress.

n = ratio of the moduli of elasticity, steel to concrete; $n = \frac{E_s}{E_c}$.
o = observed.
p = ratio of areas, steel to concrete; $p = \frac{A_s}{A_c}$.
r = contraction.
s = steel.
t = theoretical.
u = ultimate strength.
A = area; subscripts: s = steel; c = concrete.
D = inches of deformation in 8 in.
E = modulus of elasticity; subscripts: s = steel; c = concrete.
P = load in pounds; subscripts: e = estimated; d = design.
W = weight of beam and slab, in pounds per foot.
ϵ = coefficient of contraction.

General

All stresses reported in this paper may be considered due to dead loads because, even after the building was finished and opened to occupancy, the loads superimposed during the period of observation were negligible. The following equation gives the general expression for unit stress, in pounds per square inch, due to deformation:

$$f = \frac{D\,E}{5\,000 \times 8}$$

in which, D is the gauge reading or deformation in 8 in. For steel the modulus of elasticity was found to be close to 30 000 000 lb per sq. in., and, for concrete, it will be assumed to be 2 000 000 lb per sq in. at 28 days. (In this paper no attempt was made to note the variation in the modulus of elasticity of concrete over a period of time.) For steel, the unit stress, in pounds per square inch, is:

$$f_s = \frac{30\,000\,000}{5\,000 \times 8}\,D = 750\,D \qquad (1)$$

To compare the observed effects with the theoretical or calculated, care was exercised in observing the actual load on the member considered. For example, the load on a column with shoring in place is unknown; but without the shoring, the load can be computed accurately. A log of the construction was kept, and at any stage the load on any member could be estimated.

The shrinking process continues for a long time if the concrete is allowed to dry. A failure does not retard the action of the contraction. However, on account of a failure a member may cease to act as a unit. Tension failures in the concrete separate the member into smaller units, each unit contracting but not necessarily with a cumulative effect between given points. D. A. Abrams, M. Am. Soc. C. E., found that appreciably higher stresses were obtained by using 2-in. gauge lines as against 8-in. gauge lines. This is to be expected as there are fewer failures in the shorter gauge length.

From the theoretical equation for compressive stress in the steel due to shrinkage (based on the assumption that a member of constant cross-section contracts toward the center of the member) the maximum stress occurs at the center of the member.[3] The test points in the columns are located very nearly at that point, so that the observed stresses may be considered as the maximum compressive stresses in the steel.

Although concrete is an excellent building material, it is unsatisfactory from the standpoint of the theorist. That its modulus of elasticity is not constant has been proved conclusively. Strictly speaking, concrete is not an elastic material and, therefore, it can have no modulus of elasticity. However, a ratio of stress to strain can be obtained at any desired point on the stress-strain curve. Although much work has been done, it is difficult to arrive at the ratio of the moduli of steel to concrete for a given case, with a fair degree of accuracy. Constant values of n will be used in this paper for the purpose of comparison.

Disregarding the effects induced upon the steel and the concrete by shrinkage, stresses in plain reinforced columns are quite easily calculated. This is not the case for helically reinforced columns. Elastic longitudinal contraction in a column produces lateral expansion; and, conversely, elastic lateral expansion will cause longitudinal contraction. (This is the phenomenon noted by Poisson.) In a plain longitudinal reinforced concrete column, under an imposed load, the lateral expansion occurs without much restriction, as contrasted with the helically reinforced columns in which there is considerable restriction. This amounts to hindering longitudinal contraction reactively; and, since the stress is a function of the deformation, any manner of preventing elastic longitudinal contraction will strengthen the column. This fact is generally appreciated.[4] The stresses allowed by the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete for helical columns compensate for the use of the helical reinforcement, but these are not the theoretical stresses. For comparative use, let the theoretical steel stress, f''_{st}, due to a load, P, be obtained, in pounds per square inch, from the equation:

$$f''_{st} = \frac{P\,n}{A_c + (n-1)\,p\,A_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which, A_c is the area of concrete; and $p = \frac{A_s}{A_c}$.

Basement Story Columns (Concrete Mix, 1:1:2)

Experimental Results.—The experimental readings are recorded by means of graphs. Curves in every case are drawn as the mean of the average readings, and do not necessarily pass through the average points. At best, these

[3] *Bulletin No. 126*, Univ. of Illinois Eng. Experiment Station, p. 21.

[4] "Stresses in Helically Reinforced Concrete Columns," by A. F. Zesiger, M. Am. Soc. C. E., and E. J. Affeldt, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 379.

curves give the general trend of the variation in the deformations, and extreme accuracy in the individual readings can not be expected. In an investigation of this kind, there are too many non-controllable variables that affect the experimental results. However, the writer believes that the graphs do help one to appreciate the action taking place in the members under the average construction conditions.

It is seen from Fig. 2 that the unit stresses for the first five days showed a definite increase; but on the sixth day a sudden decrease occurred. At this time the beams and slab of the floor above were poured, and the increase in

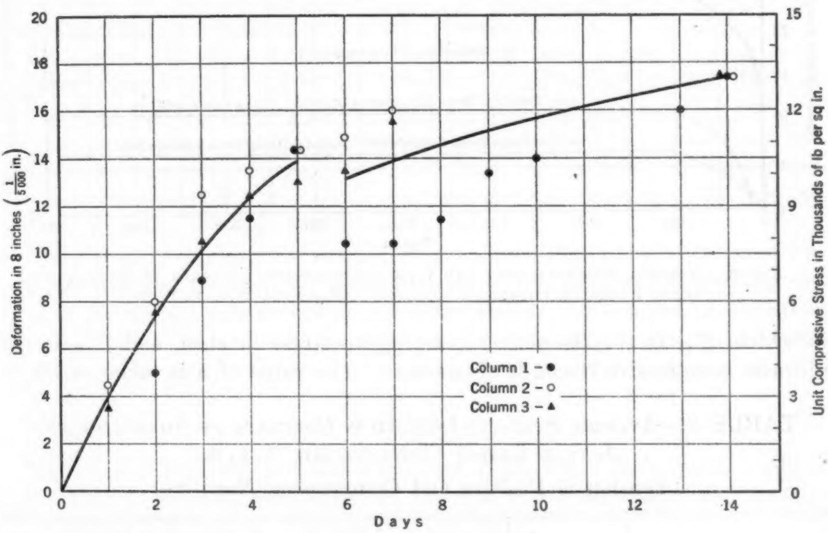

FIG. 2.—DEFORMATIONS AND UNIT COMPRESSION STRESSES OF STEEL IN WEST FACES OF BASEMENT STORY COLUMNS

load (if it did get to the columns) should have indicated an increase in the deformations. However, a definite decrease was observed. Apparently, minor failures or re-adjustments allowed the steel to return, partly, to its unstressed length. Some readers may question the occurrence of the sudden drop in the curves, and conclusive evidence can not be given to defend it. It is merely a personal interpretation of the experimental readings.

Similar curves were plotted of the readings on the east faces of the basement columns. A close scrutiny of the two curves showed them to be almost identical for the first five days and differing from that time, until at fourteen days the east-face stresses were less than the west-face stresses, as they should be. The eccentricity of loading causes this difference. The curves for the concrete columns of lower strength bring out this detail much better.

Fig. 3 shows the average readings and the mean curve for a period of observation of approximately 2 years and 4 months. There is a gradual,

almost constant, rise from 200 days to the end of the curve. The horizontal line, Curve 2, Fig. 3 (f''_{st} = 12 500 lb per sq in.) is governed by the formula,

$$f''_{st} = [300 + (0.1 + 4\,p)\,f''_{cu}]\,(n) \quad \ldots\ldots\ldots\ldots\ldots(3)$$

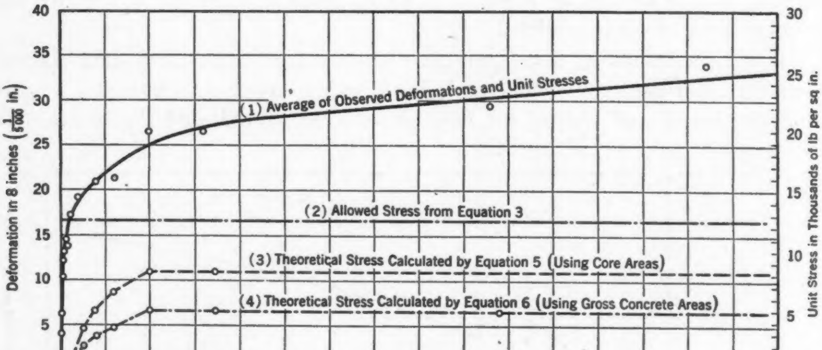

FIG. 3.—AVERAGE DEFORMATIONS AND UNIT COMPRESSIVE STRESSES IN EAST AND WEST FACES OF BASEMENT COLUMNS (CONCRETE MIX, 1:1:2).

in which, f''_{st} is the theoretical compressive stress in steel, and f''_{cu} is the ultimate compressive strength of concrete. The value of n is taken as 12.

TABLE 2.—AVERAGE STRESSES INDUCED IN CONCRETE BY SHRINKAGE AND ACTUAL LOADS; CONCRETE MIX, 1:1:2.

(Tension is Positive and Compression Negative)

Age, in days (1)	Estimated load, in pounds (2)	Observed steel stress, f''_{so}, in pounds per square inch (3)	Load on steel, $A_s f''_{so}$, in pounds (4)	Load on concrete, in pounds (5)	Concrete stress, in pounds per square inch (6)
2	2 000	5 500	34 100	+32 100	+101
3	2 000	7 500	46 400	+44 400	+139
	(21 000)			(+44 600)	(+140)
7	7 000	10 600	65 600	58 600	+184
	(44 600)			(+34 000)	(+107)
14	7 000	12 700	78 600	+71 600	+224
27	67 000	14 600	90 400	+23 400	+ 73
40	90 000	15 800	97 800	+ 7 800	+ 24
61	120 000	17 100	106 000	—14 000	— 44
173	155 000	20 000	124 000	—31 000	— 97
300	160 000	21 500	133 000	—27 000	— 85
480	160 000	22 700	140 500	—19 500	— 61
600	160 000	23 500	145 000	—15 000	— 47
700	160 000	24 300	150 000	—10 000	— 31
850	160 000	25 500	157 600	— 3 400	— 10.7

A summary of the average stresses induced in the concrete by shrinkage and actual loads is shown in Table 2. In Column (2) the values in parentheses are the total estimated loads, which in all probability did not affect the deformation readings because shoring was in place up to 28 days.

The remaining values in Column (2) are the probable effective loads. The unit stress in the concrete is a computed value obtained from:

$$P_e = A_s\, f_{so} \pm A_c\, f_c \quad \text{.....................(4)}$$

in which, P_e is the estimated load; A_s, area of steel; f_{so}, average observed stress in steel; A_c, area of concrete; and f_c, unit stress in concrete. Every quantity in Equation (4) is known, except the concrete stress. The gross area minus the steel area was used for the effective, concrete cross-section; thus, $A_c = 13 \times 25 - 6.18 = 318.8$ sq in. Although the stresses are obtained by computation, nevertheless, they are the true mean values. There is some doubt about the actual loads during the first two weeks and, consequently, some uncertainty about the true induced stresses.

By plotting the values in Column (6), Table 2, against age (Column (1)), a clearer concept is afforded of the variation in the concrete stress. For the first 50 days there is tension in the concrete, followed by a reversal, until at 200 days a maximum compressive value is reached. From that time, the stress decreases under a constant load, and the burden is gradually transferred from the concrete to the steel by the shrinkage of the concrete. The average concrete stress at 850 days is 10 lb per sq in.

Calculated Theoretical Stresses.—The calculated theoretical unit stress due to actual loads in the steel for reinforced concrete columns, disregarding the effect of the helical reinforcement, is determined by substituting in Equation (2): $A_c = 2 \times 78.54 = 157.1$; $n = 12$ (assumed constant); and, $p = 0.0392$. Thus,

$$f''_{st} = \frac{P}{18.75} \quad \text{..........................(5)}$$

Equation (5) is obtained using the core areas only. Subsequent analysis shows that it is more reasonable to use the gross concrete area, namely, 13 by 25 in. = 325 sq in. Then, $p = 0.019$, and,

$$f''_{st} = \frac{P}{32.8} \quad \text{..........................(6)}$$

Equations (1) and (2) are represented graphically in Fig. 3. To obtain the apparent shrinkage effect on the steel, subtract the theoretical from the observed stress. Thus, the stress caused by shrinkage at 800 days is either 25 000 − 8 500 = 16 500 lb per sq in., or 25 000 − 4 900 = 20 100 lb per sq in., the latter figure undoubtedly approaching the actual condition obtaining in the column.

For concrete of this strength, recognized authorities recommend values of n from 6 to 10, which would give smaller theoretical unit stresses than $n = 12$, and would result in a greater difference between the observed and the calculated theoretical unit stresses.

There is every reason to believe that the concrete outside the helical reinforcement produces its share of shrinkage effects. Curve 4, in Fig. 3, is the one to use when comparing the observed stresses with the calculated theoretical stresses. Thus, at 850 days, the observed steel stress is five times greater than the theoretical stress.

Allowed Unit Stresses.—For the helically reinforced concrete columns observed in this investigation the allowed unit stresses as specified by the Joint Committee[5] are determined by Equation (3). Thus, if $p = 0.0392$; $f_{cu} = 2\ 900$ lb per sq in. at 28 days (recommended by the Joint Committee); $E_c = 2\ 500\ 000$ lb per sq in.; $n = 12$; (by Equation (3)) $f''_c = 1\ 045$ lb per sq in.; and f''_s = compressive stress in steel $= (n)\ (f_c) = (12)\ (1\ 045) = 12\ 550$ lb per sq in.

Furthermore,[5] the design load, P_d, on a column is computed by,

$$P_d = f''_c\ (A_c - A_s) + f''_s\ A_s \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

or, $P_d = 1\ 045\ (157.1 - 6.18) + 12\ 550 \times 6.18 = 235\ 200$ lb. By comparison, the estimated dead load plus live load equals 230 000 lb.

There is no correlation between the observed and the allowed steel stresses. The load on the columns at 850 days is about 70% of the design load; nevertheless, the observed, is twice the allowed, stress. Were the full design load imposed on the columns the ratio probably would be greater than 70 per cent. With an increase in the load, the concrete would be stressed more, but not to the extent of 1 045 lb per sq in. (See discussion on "First-Story Columns.")

From the foregoing remarks, it must not be concluded that the steel is unsafe when stressed in compression beyond 12 550 lb per sq in. A better working stress to use would be 20 000 lb per sq in. The steel in a reinforced concrete column if kept from buckling, should be good for a stress of 80 000 lb per sq in., and more. However, when it is stressed beyond the elastic limit, excessive deformations will result with practically no increase in the load. It is then that the concrete takes its share of the load. The allowed stress is not an indicator of the actual stresses obtaining in the columns.

First-Story Columns (Concrete Mix, 1 : $2\frac{1}{4}$: $3\frac{3}{4}$)

Experimental Results.—At the end of the fifth day, after pouring, a re-adjustment occurred in the columns composed of the richer concrete. An inspection of Fig. 4 reveals that a similar action had taken place for these columns one day later. Corresponding curves plotted from readings on the west faces show the same action. This re-adjustment may have been caused by one, or by a combination, of the following: (1) Failure of concrete in tension; (2) failure in bond at the end of the rods; (3) failure in shear between the inner core and the outer hollow cylinder containing the reinforcing steel; (4) flow of the concrete around the deformed rods; and, (5) flow of the inner core of concrete. Whether or not other re-adjustments had occurred, it is difficult to say because readings were taken at too great intervals of time. However, it seems logical to expect large flow effects in the early stage of construction and tension failures after the concrete has taken a set.

All beams under observation were designed to have no end restraint. Curves corresponding to Fig. 4, for the west faces of the columns, show the

[5] *Proceedings*, Am. Soc. C. E., October, 1924, Papers and Discussions, p. 1204, Equation (43).

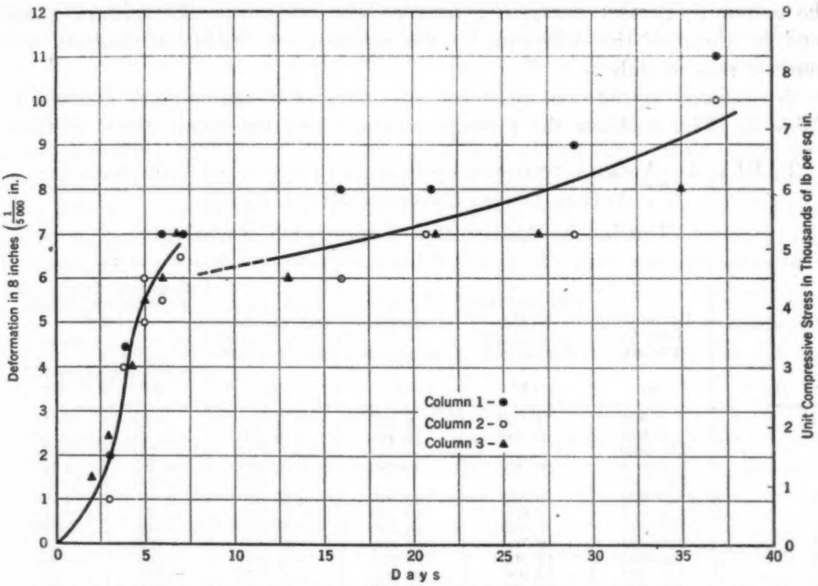

FIG. 4.—DEFORMATIONS AND UNIT COMPRESSIVE STRESSES OF STEEL IN EAST FACES OF FIRST-STORY COLUMNS.

same characteristic re-adjustment. A comparison of the stresses, however, at 28 days, shows that those on the west faces are 1 200 lb per sq in. more than on the east faces. This difference can be attributed to the eccentric loading on the columns, rather than to the continuity between the beam and

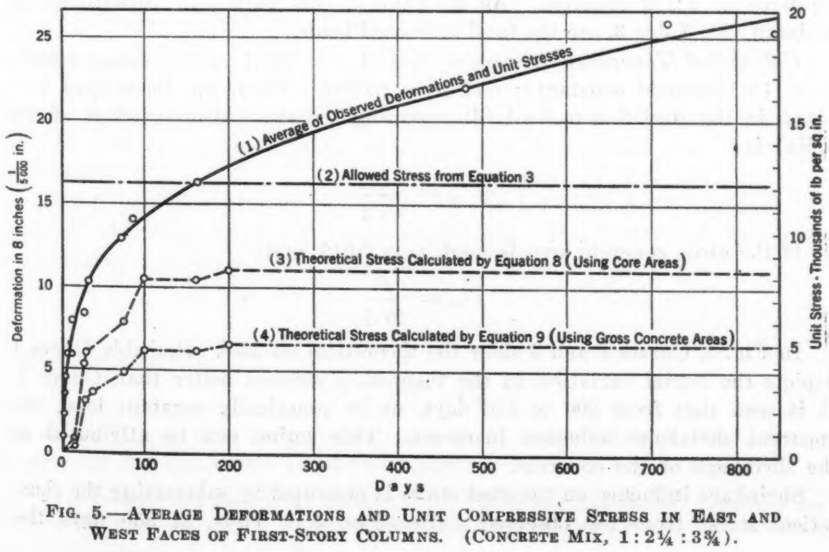

FIG. 5.—AVERAGE DEFORMATIONS AND UNIT COMPRESSIVE STRESS IN EAST AND WEST FACES OF FIRST-STORY COLUMNS. (CONCRETE MIX, 1 : 2¼ : 3¾).

the column. Perfect continuity between the beam and the column would tend to diminish the difference in the stresses. A deflection diagram will indicate this roughly.

A summary of the true mean concrete stresses in the columns is given in Table 3. Fig. 5 shows the average readings and the mean curve of these

TABLE 3.—AVERAGE STRESSES INDUCED IN CONCRETE BY SHRINKAGE AND ACTUAL LOADS; CONCRETE MIX, $1:2\frac{1}{2}:3\frac{3}{4}$

(Tension is Positive and Compression Negative)

Age, in days (1)	Estimated load, in pounds (2)	Observed steel stress, f''_{so}, in pounds per square inch (3)	Load on steel, $A_s f''_{so}$ in pounds (4)	Load on concrete (Column (2) — Column (3)), in pounds (5)	Concrete Stress, in Pounds per Square Inch	
					$A_c = 318.8$ (6)	$A_c = 157.1$ (7)
3	2 300	1 500	9 300	+ 7 000	+22	+ 45
4	7 000	2 600	16 100	+ 9 100	+ 29	+ 59
	(21 500)			(—5 100)	(—16)	(—32)
7	7 000	4 300	26 600	+19 600	+ 62	+126
	(44 000)			(—8 100)	(—26)	(—53)
14	10 000	5 800	35 900	+25 900	+ 81	+164
27	66 000	7 000	43 300	—22 700	— 71	—144
36	74 000	7 400	46 000	—28 000	— 88	—178
74	100 000	9 700	60 000	—40 000	—126	—255
160	128 000	12 300	76 000	—52 000	—163	—330
200	136 000	12 900	80 000	—56 000	—176	—357
300	136 000	14 500	90 000	—46 000	—145	—294
400	136 000	15 700	97 000	—39 000	—122	—250
480	136 000	16 600	103 000	—33 000	—104	—211
600	136 000	17 700	110 000	—26 000	— 82	—166
700	136 000	18 500	115 000	—21 000	— 66	—134
850	136 000	19 600	121 000	—15 000	— 47	— 95

stresses corresponding to Fig. 3. It must be noted that the period of induced tension in these columns is smaller in duration and magnitude than for the columns of 1:1:2 concrete. As in Table 2, the values in parentheses in Column (2), Table 3, are the total estimated loads.

Calculated Theoretical Stresses.—Let $A_c = 157.1$ sq in. (core areas); $n = 15$ (assumed constant); and $p = 0.0392$. Then, the theoretical unit stress in the steel due to load (disregarding the strengthening effect of the helix) is:

$$f''_{st} = \frac{P}{16.2}$$

If the gross concrete area is used, $p = 0.019$, and:

$$f''_{st} = \frac{P}{27.4}$$

In Fig. 5, Curves 3 and 4 show the theoretical stresses. Probably Curve 4 depicts the actual variation in the theoretical stresses better than Curve 3. It is seen that from 200 to 850 days, under practically constant load, the apparent shrinkage influence increases. This action can be attributed to the shrinkage of the concrete.

Shrinkage influence on the steel stress is measured by subtracting the theoretical stress from the observed. (See Fig. 5.) Thus, at 850 days the

apparent shrinkage influence is 19 500 − 8 400 = 11 100 lb per sq in., or 19 500 − 5 000 = 14 500 lb per sq in., which is considerably less than the 16 500 and 20 100 lb per sq in., respectively, for the columns of rich concrete mixes.

The only reason for using the 1:1:2 concrete in the basement columns was to utilize its greater strength. The Joint Committee's equations[6] for the steel stress for both mixes, in this case, give almost the same result, but the observed steel stress at 850 days for the rich mix is 6 000 lb per sq in. more than for the leaner mix. The allowed concrete stresses are 1 045 and 814 lb per sq in. for the 1:1:2 and $1:2\frac{1}{4}:3\frac{3}{4}$ mixes, respectively. An examination of values in Column (6) of Table 2, and Columns (6) and (7) of Table 3 shows that at no time do the actual stresses approach these values. Furthermore, the rich concrete, except for the tensile load imposed upon it, carries a smaller part of the actual load than the leaner mix. Obviously, the rich mix defeats the purpose for which it was intended.

Allowed Unit Stresses.—Let $p = 0.0392$, $f_{cu} = 2\ 000$ lb per sq in. (recommended by the Joint Committee); and, $n = 15$ (assumed constant). Then, for helically reinforced concrete columns the allowed unit stress in the concrete (by Equation (3)) is $f''_c = 814$ lb. per sq in. For steel, the stress is: $f''_s = nf''_c = 12\ 200$ lb per sq in.

As in the case of the basement columns, the design load is computed by Equation (7), so that $P_d = 814\ (157.1 - 6.18) + 12\ 200 \times 6.18 = 198\ 500$ lb.

There is no correlation between the observed and the allowed steel stresses. At 850 days, the imposed load is approximately 70% of the design load. The probable unit steel stress under the full design load at 850 days, using the shrinkage effect of 14 500 lb per sq in., would be:

$$f''_s = f''_r + \frac{5\,000}{70\%} \qquad (8)$$

or, $f_s = 14\ 500 + 7\ 500 = 21\ 650$ lb per sq in.

The probable unit concrete stress equals $f''_c = 50 + \frac{7\,150 - 5\,000}{15}$, or 229 lb per sq in. Apparently, therefore, the concrete strength would be utilized with an increase in the load, but the steel would show a greater over-stress.

Shrinkage.—The aim in the following computations is to compare the observed shrinkage effects with the theoretical. The theoretical maximum compressive steel stress due to shrinkage is:

$$f''_{sr} = \frac{3\ E_s\,\epsilon}{2\,(pn + 1)} \qquad (9)$$

and in the concrete:[7]

$$f'_{cr} = f''_{sr}\ (p) \qquad (10)$$

in which, f''_{sr} = unit compressive stress in steel due to contraction of concrete; and ϵ is the coefficient of contraction.

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[6] *Proceedings*, Am. Soc. C. E., October, 1924, Papers and Discussions, p. 1204 *et seq.*
[7] *Bulletin No. 126*, Univ. of Illinois Eng. Experiment Station, p. 21.

From his experiments with limestone concrete, F. R. McMillan, M. Am. Soc. C. E., deduces that the average coefficient of contraction for plain air-cured 1:2:4 concrete at 30 days is close to 0.00025 in. per in.; at 60 days, it is 0.00050 in. per in.; and, at 90 days, it is 0.00055 in. per in. For all practical purposes the latter is the maximum coefficient.[8] According to tests performed by R. E. Davis,[9] M. Am. Soc. C. E., 1:2:3 gravel concrete at the age of three months, gives about twice as much contraction as limestone concrete, other factors being constant.

An inspection of Fig. 6 shows that the deformations at the tops of the beams (where contraction is almost unrestricted), caused by shrinkage and

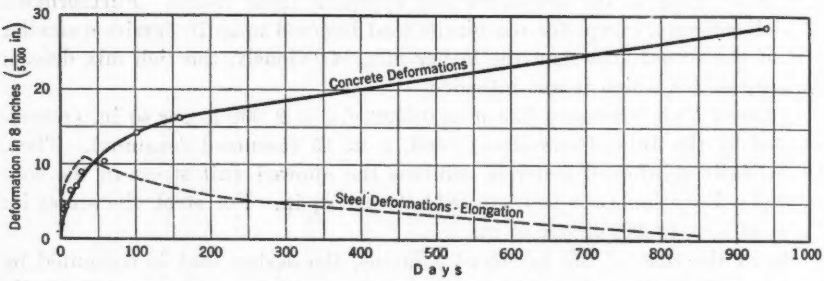

FIG. 6.—AVERAGE DEFORMATIONS OF CONCRETE AT TOPS OF FLOOR-BEAMS AND OF STEEL IN THE BOTTOMS OF FLOOR-BEAMS (CONCRETE MIX, 1 : 2¼ : 3¾).

by dead load, more nearly approach Mr. McMillan's figures. These will be used in the following computations. At 36 days, ϵ is approximately 0.025 per cent. Using the net area of the concrete as 325 − 6.18 = 318.8 sq in., $p = 0.019$, and $n = 15$: f''_{sr} (by Equation (9)) = 8 750 lb per sq in. (compression); and, f'_c (by Equation (10)) = 167 lb per sq in. (tension). At 74 days, $\epsilon = 0.05\%$; $f''_{sr} = 17\ 500$ lb per sq in.; and $f'_{cr} = 334$ lb per sq in. Other values are computed for different ages of concrete, and for $p = 0.019$, and $p = 0.0392$.

The result of these computations is recorded in Columns (9) and (10) Table 4. A discussion of the results in this table is desirable. To obtain values for the unit stress in the concrete to compare with those in Columns (6) and (7), Table 3, the following procedure was used. Subtracting the theoretical steel stress (Column (4)), from the observed steel stress (Column (3)), gave the apparent stress due to shrinkage (Column (6)). According to theory, the tensile stress in the concrete induced by shrinkage (Column (7)) is the product of Columns (6) and (13). The actual stress in the concrete (Column (8)) is the algebraic sum of the theoretical and the induced stresses. This procedure is repeated for different ages and for two values of p. Using the theoretical stresses induced by shrinkage and the theoretical stresses due to loads, the values in Column (11) are obtained. Comparing the magnitude and character of the concrete stresses with those shown in Columns (6) and (7), Table 3, for the corresponding ages, it is noted that the values in

[8] *Transactions*, Am. Soc. C. E., Vol. LXXX (December, 1916), p. 1740.
[9] *Journal*, Am. Concrete Inst., February, 1930, p. 419.

Column (8) for $p = 0.019$ and $p = 0.0392$ check favorably, the values for $p = 0.019$ giving the closer agreement. That the theoretical shrinkage stresses added to the theoretical stresses in the concrete due to load, do not agree with the values in Column (6) of Table 3, might have been anticipated

TABLE 4.—COMPUTATIONS FOR TENSILE STRENGTH OF 1 : $2\frac{1}{4}$: $3\frac{3}{4}$ CONCRETE AND COMPRESSIVE STRESS IN STEEL DUE TO CONTRACTION OF CONCRETE.

(Tension is positive, and compression is negative)

Age, in days (1)	Estimated load, in pounds (2)	Observed steel stress, in pounds per square inch (3)	Theoretical compressive steel stress due to load, in pounds per square inch (4)	Theoretical compressive concrete stress due to load, in pounds per square inch (5)	Apparent steel stress due to shrinkage, in pounds per square inch (6)	Apparent concrete stress due to shrinkage, in pounds per square inch (7)	Actual concrete stress, in pounds per square inch (8)	Thoeretical compressive steel stress, due to shrinkage, in pounds per square inch (9)	Theoretical tensile concrete stress, due to shrinkage, in pounds per square inch (10)	Actual concrete stress, in pounds per square inch (11)	Coefficient of contraction assumed (12)	p (13)
FROM OBSERVED READINGS								THEORETICAL				
36	74 000	-7 400	-2 700	-180	-4 700	+ 90	- 90	-8 750	+167	- 13	0.00025	0.019
74	100 000	-9 700	-3 750	-245	-6 050	+115	-130	-17 500	+334	+ 89	0.00050	0.019
160	128 000	-12 300	-4 680	-312	-7 620	+145	-167	-19 250	+366	+ 54	0.00055	0.019
850	136 000	-19 600	-4 970	-330	-14 630	+288	- 50	-19 250	+366	+ 36	0.00055	0.019
36	74 000	-7 400	-4 570	-305	-2 830	+110	-195	-7 070	+277	- 28	0.00025	0.0392
74	100 000	-9 700	-6 170	-412	-3 530	+138	-274	-14 140	+554	+142	0.00050	0.0392
160	128 000	-12 300	-7 900	-527	-4 400	+173	-354	-15 600	+612	+ 85	0.00055	0.0392
850	136 000	-19 600	-8 400	-560	-11 200	+440	-120	-15 600	+612	+ 52	0.00055	0.0392

since the "slips" between the steel and the concrete in the early period of construction were neglected in Column (9); but it is surprising to find such close correlation between Column (6), Table 3, and the upper half of Column (8), Table 4, despite the use of the uncertain value of n which appears only in one (Table 4) of the two groups of stresses compared.

EXPERIMENTAL RESULTS, T-BEAMS (CONCRETE MIX, 1 : $2\frac{1}{4}$: $3\frac{3}{4}$)

Steel in Beams.—A high tensile stress was developed in the first three days (under no load), dropping off considerably on the fourth day. (See Fig. 7.) The drop is larger than for the columns of the same mix. The difference may be attributed to the large size of rods used, which fail much easier in bond than the smaller rods for the same effect per square inch of cross-section.

From 7 to 30 days, the increase in the stress is quite rapid and fluctuating, giving indication of possible re-adjustments. Although shoring was removed during this time, no increase in deformations was noticed. In the period from 30 to 100 days, a large decrease was observed under constant load. (See Fig. 8.) The apparent shrinkage effect curve has the same general shape as the

column curve for the same mix, showing that variation in stress is due to shrinkage.

The curve in Fig. 8 shows the variation of the steel stress for a period of 2 years and 4 months. Under a constant load, the steel is being gradu-

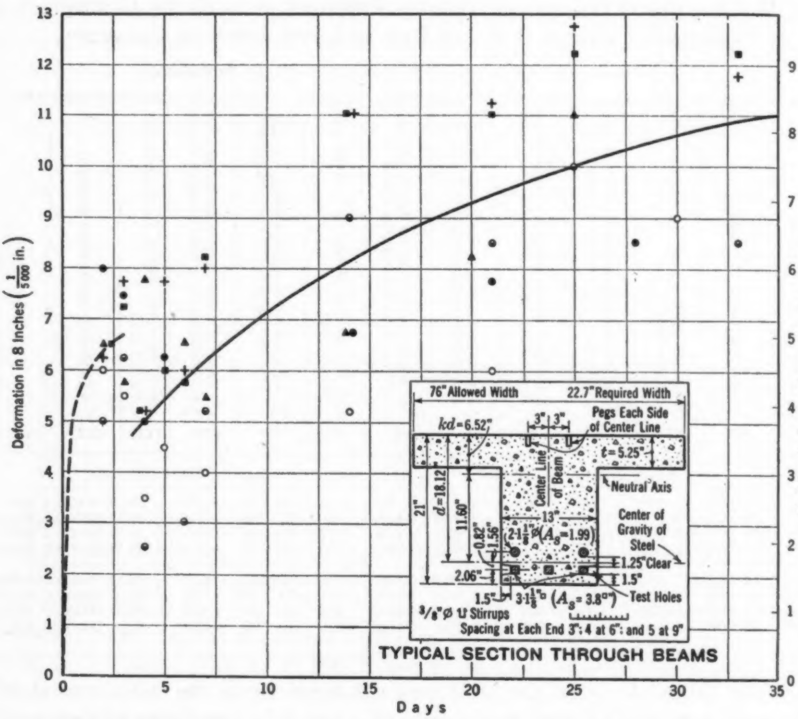

FIG. 7.—ELONGATIONS AND UNIT TENSILE STRESSES IN THE REINFORCEMENT OF THE MAIN BEAMS OF THE FIRST AND SECOND FLOORS. (CONCRETE MIX, 1 : 2¼ : 3¾).

ally contracted. The average tensile stress at 850 days was obtained from twelve readings, some of which indicated compressive stresses. It may be expected that in the future compressive stresses in the steel may result, unless a re-adjustment occurs caused by a tension failure in the concrete.

To obtain an approximate measure of the effect of shrinkage on the beam steel at 850 days, it seems reasonable to use:

$$\left(\begin{array}{c}\text{Shrinkage}\\ \text{stress}\end{array}\right) = \left(\begin{array}{c}\text{Calculated unit}\\ \text{stress}\end{array}\right) - \left(\begin{array}{c}\text{Observed unit with}\\ \text{stress at 850 days}\end{array}\right)$$

or, 9 360 lb per sq in. (compression) = 9 860 − 500.

Concrete in Beams.—Fig. 6 shows the average of the concrete deformations at the tops of the beams. The points on the curve before 160 days represent an average of twelve readings obtained from six beams, and after 160 days an average of six readings obtained from three beams. Shrinkage of

free concrete was not studied in this paper; therefore, it is not possible to obtain numerical values for the actual stresses in the concrete.

Calculated Theoretical Stresses.—The following considerations entered into the computation of stresses in the beams: (*a*) All beams under observa-

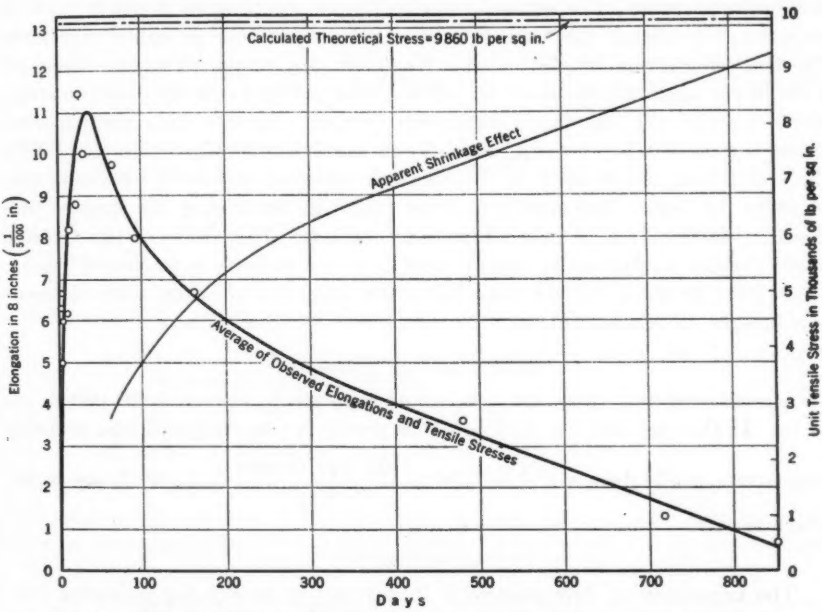

FIG. 8.—AVERAGE ELONGATIONS AND UNIT TENSILE STRESSES IN REINFORCEMENT OF MAIN BEAMS OF FIRST AND SECOND FLOORS.

tion had no theoretical end restraint; (*b*) bending moment due to uniformly distributed load $= \frac{wl^2}{8}$ ft-lb; (*c*) dead load = weight of beam + weight of slab + $\frac{3}{4}$-in. finish; (*d*) weight of beam = 205 lb per ft of beam; (*e*) weight of slab = 787 lb per ft of beam; (*f*) total weight (Item (*d*) + Item (*e*), w, = 992 lb per ft of beam; and, (*g*) bending moment = 71 500 ft-lb.

Furthermore, the Seattle Building Code allows a **T** of: b = width of beam + thickness of slab $= 13 + 2 \times 6 \times 5.25 = 26$ in.; $f_s = 20\ 000$ lb per sq in.; $f_c = 750$ lb per sq in.; $n = 15$ (assumed constant); and $\frac{t}{d} = \frac{5.25}{18.12} = 0.29$. For the foregoing stresses, $k = 0.36$; $p = 0.0065$;

$$K'' = 115;^{10}\ j = \left(1 - \frac{t}{d}\right) \frac{3\,k - 2\,\frac{t}{d}}{6\,k - 3\,\frac{t}{d}} = 0.888; \text{ and } jd = 0.888 \times 18.12 = 16.1 \text{ in.}$$

Therefore, $f'_s = \frac{M}{A_s\, jd} = 9\ 200$ lb per sq in. (calculated stress);

[10] "Structural Members and Connections," by Hool and Kinne, p. 469.

f'_s (extreme) $= 9\ 200 \dfrac{(11.60 + 0.82)}{11.60} = 9\ 860$ lb per sq in.; $Kbd^2 = M$ $= 71\ 500 \times 12$; and $b = 22.7$ in., which is the required width.

The extreme fiber stress was obtained by the ordinary design method. The approximate value of k equal to three-eighths would give a result just as accurate considering the uncertainties; and neither one probably represents the true conditions in the beam. However, for comparison, the stress of 9 860 lb per sq in. will be used. It will be noted in Fig. 8 that the observed stress never reached the calculated theoretical stress. At 850 days the observed stress is only 500 lb per sq in., much lower than is generally suspected. With the steel stress as low as it is, the question arises as to how the beam is able to carry the load. The answer is to be found in combining the stresses produced by the load and the shrinkage deformations. This phase of the problem needs further study, but it appears that it is not entirely a matter of theory. Tests must be made to determine the actual behavior of beams with different percentages of reinforcement.

Allowed Unit Stresses

The allowed unit stress for hard grade steel reinforcement is 20 000 lb per sq in. If the specified live load of 60 lb per sq ft were imposed, the probable total stress at 850 days would be: $500 + \dfrac{60\ (12)\ (9\ 860)}{992} = 8\ 660$ lb per sq in., which is safe.

Conclusions

The experimental data presented in this paper justify the following conclusions, assuming that all concrete areas in the columns, unless otherwise noted, are taken as the gross net areas; that is, 318.8 sq in.:

1.—Concrete and steel do not act together during the first seven days after pouring, whereas the usual assumption is that they act together at all times.

2.—The initial steel stresses are high. Initial stresses are understood to be those which exist in the member before the load is applied. In construction similar to this, the load can be carried by the member only when the shoring is removed, in this case at the end of 28 days.

(*a*) For the rich-mix concrete column at 28 days there is an initial compressive steel stress of 12 500 lb per sq in., which is slightly more than that recommended by the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete for the maximum design stress in a column of this type and for this percentage of longitudinal reinforcement.

(*b*) The leaner-mix concrete column at 28 days shows an initial compressive steel stress of 4 500 lb per sq in., about one-third the allowed stress. Thus, the shrinkage and flow of the concrete impose an extra load on the steel for which no allowance is made in the design.

3.—The actual unit compressive stresses in the columns of rich-mix concrete at 2 years and 4 months was 25 000 lb per sq in. for the steel and 10 lb per sq in. for the concrete, whereas the allowed stresses are 12 550 lb per

sq in. and 1 045 per sq in., respectively. Accordingly, there is not even a reasonable correlation between the actual and the allowed unit stresses, although it should be noted that the imposed load is 70% of the design load. If the full design load were applied to the column, the observed stresses probably would both be greater, leaving the concrete safe and the steel greatly overloaded.

4.—The actual unit compressive stresses in the column of leaner-mix concrete at 2 years and 4 months was 19 500 lb per sq in. in the steel and 50 lb per sq in. in the concrete, the allowed stresses being 12 700 lb per sq in. and 814 lb per sq in., respectively.

5.—The actual unit tensile stress in the beam steel at 2 years and 4 months was 500 lb per sq in., whereas the allowed stress is 20 000 lb per sq in. The calculated theoretical stress in this case due to dead load alone is 9 860 lb per sq in.

6.—The influence of the shrinkage and flow of the concrete on the steel at 2 years and 4 months shows that there was:

(*a*) Nearly 20 000 lb per sq in. compressive stress developed in the rich-mix concrete column;
(*b*) Approximately 14 000 lb per sq in. compressive stress developed in the lean-mix concrete column; and,
(*c*) About 9 360 lb per sq in. compressive stress in the beam.

7.—The effect of framing a beam on one side of a column produced flexural stresses in the column. The magnitude of these stresses at 28 days was 600 lb per sq in. compression in the steel nearest the beam and 600 lb per sq in. tension in the opposite steel.

8.—No additional deformation was observed in the members when shoring was removed. It is possible that the load was gradually transferred to the members by the drying of the form lumber and shoring.

9.—Shrinkage should be considered as a property of concrete, and design equations should be altered to include the effect of shrinkage of the concrete both on the steel and on the concrete.

10.—The empirical stresses, recommended by the Joint Committee for allowed stresses in columns, give no indication of the actual stresses prevailing in the column at this early stage of loading.

It would be poor engineering to deduce general equations for beams and columns from the results recorded in this paper, because of the comparatively small number of members tested. However, these observations have shown that it is questionable to use rich concrete for columns without making due allowance for the increased compressive stress induced in the steel. In general, the shrinkage of the concrete greatly increases the stress in the steel and decreases that in the concrete. This action for the 1 : 1 : 2 concrete columns gives a steel stress at 850 days 50% higher and a concrete stress 80% lower than the corresponding stresses in the 1 : $2\frac{1}{4}$: $3\frac{3}{4}$ columns. When loaded to failure helically reinforced columns of rich concrete show greater strength than columns of leaner mixes because of the redistribution of stresses as the elastic limit of the steel is approached.

The compressive steel stress in the columns is higher than is generally suspected. It is evident that variation in the percentages of steel used in the columns will give different results. A correlated study of columns with different percentages of steel subjected to increasing load increments approximating actual conditions, is desirable. These tests should simulate the average construction conditions, particularly in the manner and medium by which the load is imposed on the column. It is important to put the load on the concrete of the column as in actual construction. Loads thus applied tend to "strip" the concrete from the steel. If steel plates are used at the ends of the column and there is contact with the longitudinal reinforcement, a condition is created which does not approach the actual, and the "stripping" factor is neglected.

From the foregoing data and remarks, it may be concluded that the average column design favors the concrete but not the steel. Based on the allowed steel stress, the designs are unsafe. The steel in the 1 : 1 : 2 columns is heavily stressed. It has been proved by tests that the shrinkage phenomenon goes on for many years; eventually, the yield point of the steel is reached resulting in large deformations under a constant load. When this occurs the concrete will take a larger share of the imposed load, a fact which may explain why some reinforced concrete columns do not fail when seemingly they should.

The beam design is quite safe for the steel and the concrete. When it is realized that shrinkage does affect the steel stress, a comprehensive study of beams should make it possible to use this factor in the design.

Future investigations should record readings at shorter intervals of time, especially in the early stage of construction. Shrinkage of unstressed, plain concrete should be recorded for comparison and study of the experimental results. This would be useful in obtaining the concrete stresses due to load at the top of the beams, and would aid in developing a design equation involving shrinkage of the concrete.

In conclusion, it may be stated that although the imposed loads on the members tested did not reach the design loads, the condition is representative of many cases met in practice. Furthermore, engineers are inclined to study the strength of reinforced concrete structures under breaking loads, and to slight their behavior under actual conditions. It is hoped that, in the future, tests on the beams discussed in this paper, will be made with the full design loads.

Acknowledgments

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PAPERS

INTERCEPTING SEWERS AND STORM STAND-BY TANKS AT COLUMBUS, OHIO

BY JOHN H. GREGORY[1], R. H. SIMPSON[2], ORRIS BONNEY[3], AND ROBERT A. ALLTON[4], MEMBERS, AM. SOC. C. E.

SYNOPSIS

The sewerage system of a city is designed, in general, to meet certain assumed conditions with the thought that when these conditions have been reached, a new plan will be developed to provide additional facilities.

In 1908, the City of Columbus, Ohio, completed comprehensive sewerage improvements designed to give adequate facilities for some years to come. The time arrived when the capacity of these works was reached or exceeded, and it became necessary to provide additional facilities not only to give adequate sewerage to the city but also to meet higher sanitary standards. A program was adopted, therefore, designed to give adequate facilities for estimated 1960 conditions. This paper describes that part of the program which has been completed and is now (1933) in operation.

The works described consist of (*a*) intercepting sewers with sufficient capacity so that overflows of storm sewage to the streams are limited to a predetermined number per year; (*b*) regulator chambers to control the quantity of storm sewage entering these sewers; and (*c*) storm stand-by tanks to give partial treatment to all storm sewage in the intercepting sewers over and above the quantity that can be handled by sewage treatment works.

INTRODUCTION

In 1908 the City of Columbus completed sewerage improvements[5] consisting of certain trunk sewers, sewage and storm-water pumping stations,

NOTE.—Discussion on this paper will be closed in **January, 1934**, *Proceedings*.

[1] Cons. Engr.; Prof. of Civ. and San. Eng., The Johns Hopkins Univ., Baltimore, Md.

[2] Chief Engr., Dept. of Public Service, Columbus, Ohio. Mr. Simpson died on May 18, 1933.

[3] Sewerage Relief Engr., Dept. of Public Service, Columbus, Ohio.

[4] Sewage Disposal Engr., Dept. of Public Service, Columbus, Ohio.

[5] "The Improved Water and Sewage Works of Columbus, Ohio," by John H. Gregory, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXVII (1910), p. 206.

force mains, and sewage treatment works. Since 1908 the city has enjoyed a steady growth in population and area; in 1930, the population, according to the United States Census, was 290 564 and the area, 38.8 sq miles.

With increasing population and sewage flow, the need for additional facilities was forcibly brought to the attention of the City Officials by the unsightly and polluted condition of the Olentangy and Scioto Rivers and of Alum Creek, both within and below the city (see Fig. 1). These conditions were officially brought to the attention of the Ohio Department of Health by officials and residents of counties located down stream, with the result that formal action was taken by the State of Ohio in 1927 whereby the City was ordered "* * * to install such works or means as may be necessary for collecting and disposing of the sewage of the City in a manner to correct and prevent the pollution of the Scioto River and Alum Creek * * *." The Olentangy River is tributary to the Scioto near the center of the city.

The general plan adopted in compliance with this order involves the construction of intercepting and sanitary sewers, regulator chambers on combined sewers, the conversion of certain districts sewered on the combined system to districts sewered on the separate system, the construction of storm stand-by tanks on the Scioto River and on Alum Creek, and the construction of additional sewage treatment facilities.

As a part of the general plan, the City has constructed, since 1927, about eight miles of intercepting sewers, eight regulator chambers for intercepting predetermined volumes of flow from combined sewers, storm stand-by tanks at Whittier Street, on the Scioto River, and a similar tank at East Main Street, on Alum Creek, all of which have been placed in service, and it is the purpose of this paper to describe the design and construction of these works.

Sewer Districts and Collection of Sewage

The city, and adjacent territory in Franklin County, is divided naturally into the three main sewer districts shown in Fig. 1, the East Side District, the Intercepting Sewer District, and the West Side District.

East Side District.—Originally, the East Side District was provided with sewers on the combined system (sanitary and storm), but in recent years extensions have been made on the separate system. The principal outlets for the district are the 9-ft Main Street combined sewer, which had a storm overflow discharging into Alum Creek, and the 42-in. sanitary sewer, which discharges into one of the new intercepting sewers. Prior to the construction of the works described herein the dry-weather flow in the Main Street Sewer was diverted to the East Side Sewage Pumping Station and from there pumped over a divide into the Intercepting Sewer District.

There were two pumping stations—on Nelson Road and Industrial Avenue—from which small quantities of sewage were pumped into sewers tributary to the Main Street Sewer. The East Side Sewage and Industrial Avenue Pumping Stations were shut down so that the sewage would flow by gravity from the East Side District through a new sewer to the Intercepting Sewer

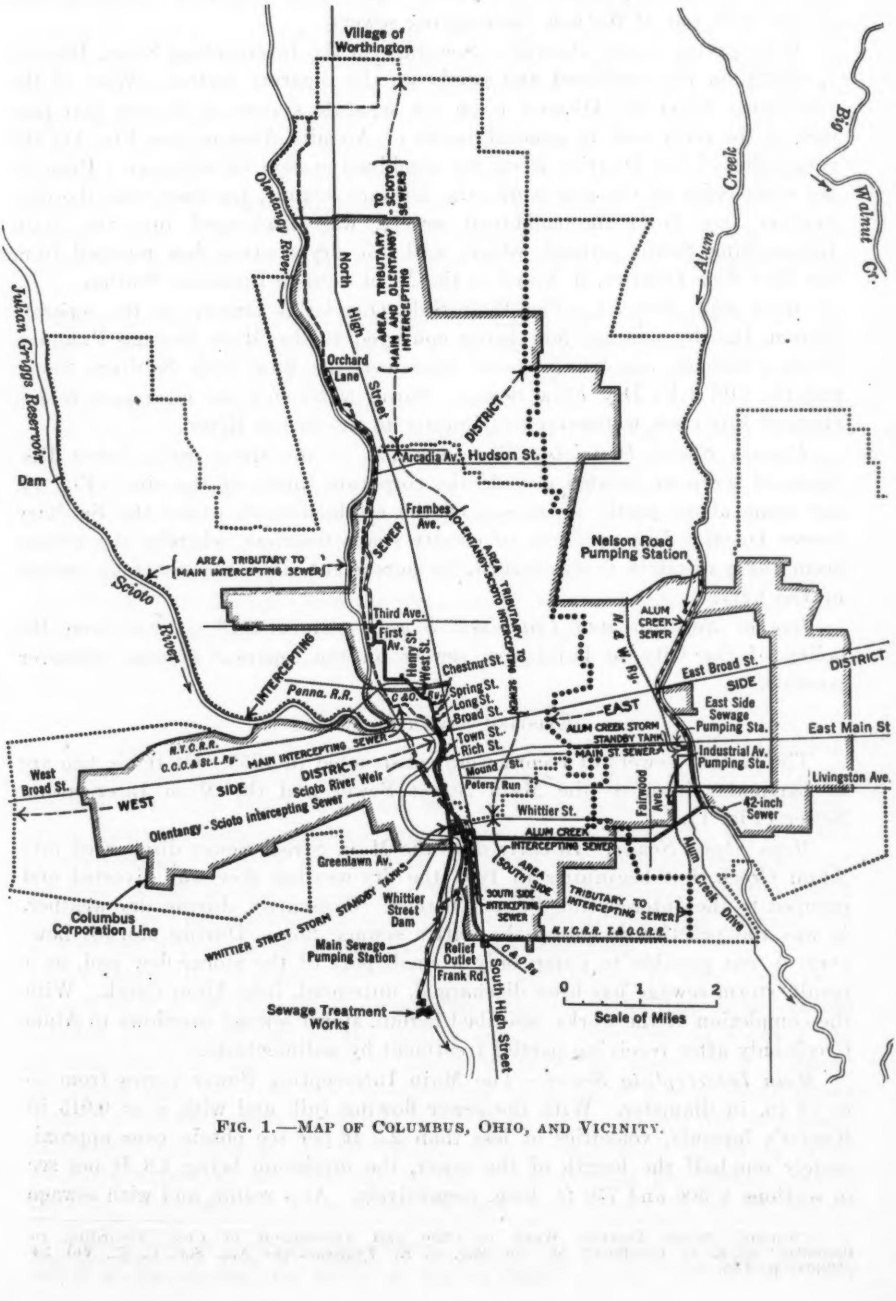

FIG. 1.—MAP OF COLUMBUS, OHIO, AND VICINITY.

District. The Nelson Road Pumping Station will remain in service pending the construction of a sewer to connect the small area now served by this station with one of the new intercepting sewers.

Intercepting Sewer District.—Sewerage in the Intercepting Sewer District is partly on the combined and partly on the separate system. West of the Olentangy River the District is on the separate system, as is also that part east of the river and, in general, north of Arcadia Avenue (see Fig. 1); the remainder of the District is on the combined system of sewerage. Prior to the completion of the new works the sanitary sewage, together with the dry-weather flow from the combined sewers, was discharged into the Main Intercepting Sewer through which, with the dry-weather flow pumped from the East Side District, it flowed to the Main Sewage Pumping Station.

West Side District.—The West Side District is largely on the separate system, the dry-weather flow being conveyed to the Main Sewage Pumping Station through two trunk sewers[5] known as the West Side Sanitary Sewer and the Old 4-Ft Dry Flow Sewer. Storm water, for the most part is discharged into open watercourses tributary to the Scioto River.

County Sewer Districts.—The inclusion in the three main Sewer Districts of territory located outside the corporate limits of the city (Fig. 1), has come about partly by reason of the establishment, under the Sanitary Sewer District Law of Ohio, of county sewer districts[6] whereby the sewage from these districts is discharged, by agreement, into the sewerage system of the City.

Use of Separate and Combined Sewers.—Since 1923, it has been the policy of the City to build new sewers on the separate system, wherever possible.

Existing Sewerage Works

The earliest sewers of record were constructed in 1853. Of these, two are of particular interest—the Main Street Sewer and the Main Intercepting Sewer (Fig. 1).

Main Street Sewer.—In early days the Main Street Sewer discharged into Alum Creek, but beginning in 1908 the dry-weather flow was diverted and pumped to the Intercepting Sewer District. Ordinarily, during dry weather, it was the practice to pump the entire sewage flow. During storms, however, it was possible to pump only a small part of the storm flow and, as a result, storm sewage has been discharged, untreated, into Alum Creek. With the completion of the works described herein, storm sewage overflows to Alum Creek only after receiving partial treatment by sedimentation.

Main Intercepting Sewer.—The Main Intercepting Sewer varies from 15 to 72 in. in diameter. With the sewer flowing full, and with $n = 0.015$ in Kutter's formula, velocities of less than 2.5 ft per sec obtain over approximately one-half the length of the sewer, the minimum being 1.8 ft per sec in sections 1 500 and 750 ft long, respectively. As a result, and with sewage

[6] "County Sewer District Work in Ohio and Assessment of Cost According to Benefits," by E. G. Bradbury, M. Am. Soc. C. E., *Transactions* Am. Soc. C. E., Vol. 94 (1930), p. 445.

from combined sewers, deposits have occurred which have had to be removed at intervals. In the future this line will be a sanitary sewer, and one important reason for converting it was the low velocities obtaining; furthermore, the sewer has been found to be of inadequate capacity at various points. Most of the connectons between the Main Intercepting Sewer and the combined sewers were small pipes varying from 6 to 15 in. in diameter and extending from sumps, 1 ft or 2 ft deep, in the bottom of the combined sewers, to the intercepting sewer. In general, there were no means of closing these connections, or of controlling the volume of flow through them. At three connections, sand-catchers had been constructed in order to decrease the quantity of sand and grit entering the intercepting sewer.

Main Sewage Pumping Station.—Normally, in dry weather, all the sewage flowed to the Main Sewage Pumping Station from which it was pumped to the sewage treatment works. During periods of high water in the Scioto River, however, sewage was pumped either to the treatment works, or to the river, and, at times, it overflowed to the river through a relief outlet on the Main Intercepting Sewer, near the pumping station. During 1932 the average volume pumped was 36 000 000 gal daily. All the sewage, however, was not pumped, and it is estimated that actually the sewage flow was about 37 000 000 gal daily.

Sewage Treatment Works.—The sewage treatment works, shown in Fig. 1, are of the Imhoff tank-trickling filter type. For a number of years the works have been greatly overloaded, with the result that the treatment afforded has been insufficient to produce a satisfactory effluent.

Flow of Rivers and Streams

By reference to Fig. 1, it will be seen that two rivers, the Olentangy and the Scioto, and a smaller stream, Alum Creek, pass through the city.[7] In the summer and early fall months, the flow in these streams is usually small, often, however, attaining flood proportions in the late fall, winter, and spring months. During the summer and early fall months, practically all the flow of the Scioto is taken for water-supply purposes, with the result that the flow through the central part of the city during such periods consists of but little more than the run-off of the Olentangy.

As illustrative of stream flows, it may be stated that, in 1930, there were periods of 2, 4, 10, 20, and 31 consecutive days when the flow in the Scioto through the central part of the city, where the water-shed is about 1 610 sq miles, was less than 4, 12, 20, 36, and 48 cu ft per sec, respectively. During periods of such low flows the corresponding velocities of flow at West Broad Street, in the pool above the Scioto River Weir, do not exceed 0.02 ft per sec. Ordinarily, the flow in the Olentangy, with a water-shed of 536 sq miles, would be somewhat less than that of the Scioto through the central part of the city. Similarly, on Alum Creek, in 1930, there were 3, 11, 16, 27, and 62

[7] For a map of Columbus and adjacent water-sheds, see "The O'Shaughnessy Dam and Reservoir," by John H. Gregory, C. B. Hoover, and C. B. Cornell, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1432.

consecutive days when the flow at Livingston Avenue, where the water-shed is 190 sq miles, was less than 4, 5, 10, 14, and 19 cu ft per sec, respectively, with corresponding velocities of flow in the stream varying from 0.2 to 0.5 ft per sec.

These values emphasize the fact, that, with such low flows and velocities, only a little sewage, even if diluted with storm water, could be discharged into the streams, in warm weather, as they flow through Columbus, without creating objectionable conditions.

Overflow of Sewage to Rivers and Streams

From the combined sewers in the Intercepting Sewer District there were nineteen storm overflow outlets to the Olentangy and Scioto Rivers. These combined sewers flow in general in westerly directions toward the two rivers and range in size, at the overflow outlets, from 24 in. to 10 ft 6 in. in diameter.

In times of storm, the volume of flow passing through the intercepting connections to the Main Intercepting Sewer was limited by the capacity of the intercepting sewer and by the size of the connections. As a result, storm sewage overflowed to the rivers and it has been estimated that there were about thirty-five such overflows per year from each of the combined sewers. Most of the intercepting connections, built many years ago, were of a type easily clogged, and, following storms, many of them were found to be completely closed, at frequent intervals, thus permitting sewage to overflow until the connections were cleaned.

The Main Intercepting Sewer was barely of sufficient capacity to handle the average rate of dry-weather sewage flow. With a maximum rate of flow during certain hours of the day, and at certain points along the sewer, sewage, unable to enter the sewer already flowing full, and at times under pressure, overflowed to the rivers. Furthermore, as already stated, sewage entered the Scioto River, at times, through the Relief Outlet.

In the East Side Sewer District, in addition to the outlet near East Main Street, there were three outlets north of this point from which overflows occurred, but during storms only.

Pollution of Rivers and Streams

As stated previously, in a number of places the sewage or storm sewage could overflow to the streams. As a result, unsightly conditions and offensive odors were caused by the stranding, sedimentation, and subsequent putrefaction of heavy solids and floating materials which enter the streams during such periods of overflow. In addition, offensive conditions in the Scioto River below the sewage treatment works were brought about by the lack of capacity of the works to treat, satisfactorily, all the dry-weather sewage flow.

As a result of an investigation of stream conditions in and near Columbus, the Division of Sanitary Engineering of the Ohio Department of Health submitted a report on this subject, in 1927, to the officials of the State Health

Department. The following brief summaries, based on statements made in this report, will serve to give a general idea of stream conditions as observed by the Division at the time of its investigation.

Olentangy River.—Above the city, both the Olentangy and the Scioto Rivers were in a normal condition and were not polluted. Below the northerly limits of the city, however, the effect of storm overflows from combined sewers was quite marked, particularly in the lowering of the dissolved oxygen content of the river water.

Scioto River.—Through the central part of the city, and as far south as Frank Road, storm overflows from the combined sewers, and from the Relief Outlet, made the Scioto River unsightly, at certain points, due to discoloration of the water and to stranded solids and oil accumulations along the banks. At other points, decomposing sludge on the bed of the river gave rise to gas ebullition and to floating masses of sludge. At Frank Road the dissolved oxygen content was low, the stream was discolored, and odors could be detected. Below Frank Road the effluent from the treatment works is discharged into the river, and it was stated that the effluent was black and odorous, and that it contained suspended matter in some quantity. The effect of the discharge of the effluent was evident at points below the works, gross pollution, with a practical absence of dissolved oxygen, extending to points 5 and 7 miles south. From Big Walnut Creek, 8 miles below the treatment works, to Big Darby Creek, 21 miles below the works, the river gradually returned to a normal condition.

Alum Creek.—Above East Main Street, Alum Creek presented a clean appearance and analytical determinations disclosed no sewage pollution. Commencing at East Main Street, however, gross pollution occurred, which was attributed to sewage entering the creek from the storm overflow at the end of the Main Street Sewer. At times, during dry weather, the dissolved oxygen content approached zero at East Main Street and also at a point $\frac{2}{3}$ mile down stream. At a point about 2 miles below East Main Street the creek began to improve in appearance, and a normal dissolved oxygen content was reached at its confluence with Big Walnut Creek, about 4 miles farther south.

Means Adopted for Relieving Insanitary Conditions in the Scioto River and in Alum Creek

The problem of correcting and preventing the pollution of the streams resolved itself into two parts, one having to do with conditions within and immediately adjacent to the city (attributed to inadequate sewer capacity), thereby resulting in sewage overflows, storm over-flows, and clogged intercepting connections, and the other, with conditions below the city, resulting mainly from insufficient treatment facilities.

In the design of works for the correction and prevention of the pollution within and immediately adjacent to the city it was decided: (1) That new sewers of adequate capacity to prevent the overflow of sewage should be provided; (2) that the greatest possible reduction, consistent with reasonable

cost, should be made in the number of storm over-flows; and (3) that a type of intercepting connection be adopted that would be as free from clogging as possible.

In the case of the Olentangy and Scioto Rivers, compliance with the foregoing requirements necessitated the design and construction of two intercepting sewers, known as the Olentangy-Scioto Intercepting Sewer and the South Side Intercepting Sewer. They were to be of capacities sufficient to handle the dry-weather flow of sewage for a reasonable period in the future, and, in addition, to have capacities for storm sewage flows considerably greater than have heretofore ordinarily been provided in intercepting sewers. Having made provision in these intercepting sewers for relatively large volumes of storm sewage, the problem of disposing of this sewage presented itself. With a maximum rate of flow of approximately 487 000 000 gal daily, there was no question but that the storm sewage should be directed to some point down stream from the central part of the city. It was obviously not feasible to attempt to handle all the flow at the treatment works; and to discharge into the river, down stream, the flow in excess of that to be handled by the treatment works would only remove to some point below the central part of the city the condition that was being corrected up stream. It was decided, therefore, that near Whittier Street, some means of treatment should be provided for the excess flow of storm sewage prior to its discharge into the Scioto River.

On Alum Creek the problem was concentrated at East Main Street to a large extent, and it was decided that here, too, the excess flow of storm sewage from the Main Street Sewer should be given some treatment before being discharged into the creek.

Year by year the sewage flow from the East Side District increased and, in 1929, following the annexation to the city of a large area east of Alum Creek, plans were made for directing sewage from the City of Bexley and a large area in Franklin County into the City System. Consequently, it was decided that the time had come to cease pumping sewage from the East Side District and to build a gravity sewer south along Alum Creek and thence west to the Olentangy-Scioto Intercepting Sewer. As a result of this decision, the new sewer, known as the Alum Creek Intercepting Sewer, was built. Sewage flows through it and is discharged into the Olentangy-Scioto Intercepting Sewer just below the storm stand-by tanks near Whittier Street. (See Fig. 1.)

The new works comprising that part of the general plan for correcting and preventing the pollution of the streams within and immediately adjacent to the city consist of:

(1) The conversion of the Main Intercepting Sewer to a sanitary sewer, to receive the flow from existing and future sanitary sewers in that District.

(2) The Olentangy-Scioto Intercepting Sewer—to intercept sewage and large volumes of storm sewage from combined sewers in the Intercepting Sewer District north of Whittier Street. This will reduce the number of

overflows to the Olentangy and Scioto Rivers, and will convey the flow through the central part of the city to storm stand-by tanks at Whittier Street, from which point a controlled volume will flow to the sewage treatment works.

(3) The South Side Intercepting Sewer—to intercept sewage and large volumes of storm sewage in the Intercepting Sewer District south of Whittier Street, thereby reducing the number of over-flows to the Scioto River. This will convey the flows to the Olentangy-Scioto Intercepting Sewer just above the storm stand-by tanks at Whittier Street.

(4) Regulator chambers—to control the flow of storm sewage from combined sewers to the Olentangy and Scioto Rivers and to the South Side Intercepting Sewers.

(5) The Whittier Street Storm Stand-By Tanks—to give partial treatment by sedimentation to storm sewage flows in the Olentangy-Scioto Intercepting Sewer in excess of controlled volumes of flow to the sewage treatment works, before discharge into the Scioto River.

(6) The Alum Creek Intercepting Sewer—to intercept sewage from the East Side District and to convey it to the Olentangy-Scioto Intercepting Sewer just below the Whittier Street tanks.

(7) The Alum Creek Sewer—to collect sanitary sewage from areas in the East Side District north of East Main Street (now on the separate system) and to convey it to the Alum Creek Intercepting Sewer just below the storm stand-by tank at East Main Street.

(8) The conversion of certain areas in the Intercepting Sewer District, and in the East Side District north of East Broad Street (now on the combined system), to the separate system, by the construction of sanitary sewers.

(9) The Alum Creek Storm Stand-By Tank—to give partial treatment by sedimentation to storm sewage flows in the Main Street Sewer in excess of controlled volumes of flow into the Alum Creek Intercepting Sewer, before discharge into Alum Creek.

General Bases of Design of New Sewerage Works

Period of Design.—It was assumed that the intercepting sewers and regulator chambers should be of capacity sufficient to serve until 1960. The Whittier Street tanks were designed for estimated conditions as of 1945, with provision for extension to meet conditions as of 1960. Because of the smaller dry-weather flow in the Main Street Sewer, and because the area tributary is largely developed at the present time, the storm stand-by tank at Alum Creek was designed for estimated conditions as of 1960. However, provision was made for another tank should it prove desirable later.

Area Assumed to Contribute Sewage in 1960.—To relieve insanitary conditions in the streams involves the handling of sewage from areas immediately adjacent to the city as well as from the city proper. In assuming the area to contribute sewage in 1960 (Fig. 1), consideration was given to known factors, recognizing, however, that possible changes in industrial and

economic conditions, as well as other factors, may effect the boundaries of the area. The total area assumed was 86.6 sq miles, subdivided as follows:

	Square miles
Area tributary to both the Main and Olentangy-Scioto Intercepting Sewers	14.2
Area tributary to Main Intercepting Sewer	12.1
Area tributary to Olentangy-Scioto Intercepting Sewer	10.9
East Side District	33.0
Area tributary to South Side Intercepting Sewer	4.9
West Side District	11.5
Total	86.6

Population.—A house-to-house canvass of a number of typical blocks in various sections of the city was made and data were obtained as to the density of population. With these data as a guide, and with the U. S. Census records, it was estimated that the population on the area assumed to contribute sewage in 1945 will be 484 000, and that, in 1960, it will be 643 000. The distribution of population in 1945, and in 1960, is shown in Table 1(*a*). In distributing the 1960 population, densities varying from 5 persons per acre in suburban areas, to 33 persons per acre in the most built-up sections of the city, were used.

TABLE 1.—Basic Estimates by Sewer Districts, in 1945 and 1960

Year	East Side District: Alum Creek Intercepting Sewer	Intercepting Sewer District				West Side District	Total, Columns (2), (6), and (7)
		Main intercepting sewer	Olentangy-Scioto Intercepting Sewer	South Side Intercepting Sewer	Total, Columns (3), (4), and (5)		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(*a*) Population							
1945	108 500	65 200	179 970	46 730	291 900	83 600	484 000
1960	157 000	108 600	209 800	55 600	374 000	112 000	643 000
(*b*) Average Dry-Weather Sewage Flow, in Million Gallons Daily							
1945	10.9	6.4	30.0	4.2	40.6	81.1	59.6
1960	15.7	10.9	36.6	5.6	53.1	11.2	80.0

Sewage Flow.—After studying the records of sewage pumped at the Main Sewage Pumping Station, it was assumed that the average sewage flow from the area contributing sewage in 1960 would be 125 gal per capita per day, or a total of about 80 000 000 gal daily. In residential districts on the separate system of sewerage a flow of 100 gal per capita daily was assumed, whereas, in districts on the combined system, within which are located most of the commercial and industrial developments, the flow was assumed at 185 gal per capita daily.

In the design of the sewers, maximum rates of sewage flow were assumed as never less than twice the average rate, and for small districts as great as four times the average rate of flow, as shown on a curve[8] developed for the

[8] "Two Diagrams for Use in Designing Sanitary Sewers," by James R. McComas, *Engineering News-Record*, Vol. 96, (June 10, 1926), p. 951.

Metropolitan Sewerage Commission of New York City in 1910 by one of the writers. The estimated average dry-weather sewage flows in 1945 and in 1960 are shown in Table 1(*b*).

Storm-Water Flow.—In the design of intercepting sewers it has been common practice to make provision for some storm-water to care for the first flushings of streets and sewers. Such provision has usually been made by adding an arbitrary quantity to the assumed rate of sewage flow per capita.

In the design of the Olentangy-Scioto and South Side Intercepting Sewers, however, one of the governing conditions was that the greatest possible reduction, consistent with reasonable cost, be made in the number of storm over-flows from the combined sewers tributary to these intercepting sewers, thus making it necessary to provide for considerably greater volumes of storm-water than have usually been handled in intercepting sewers.

Inasmuch as the design of these two sewers was largely a problem of storm-water design, the so-called rational method for estimating storm-water flow was used. In the application of this method the following values of the run-off coefficient, c, were, in general, assumed: 0.75 for commercial areas; 0.45 for industrial areas; 0.35 for residential areas; and 0.15 for parks and railroad yards. The inlet time was taken at 10 min.

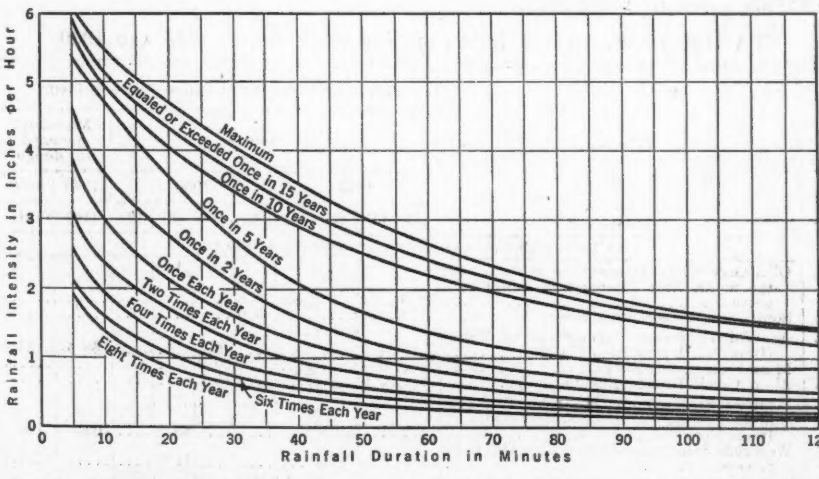

FIG. 2.—INTENSITY, DURATION, AND FREQUENCY RAINFALL CURVES FOR COLUMBUS, OHIO.

Rainfall intensities in Columbus were computed, for different durations and frequencies, from records of the United States Weather Bureau for the years, 1897 to 1923, inclusive. The curves of Fig. 2, were developed from a study of the rainfall intensity.

Studies and estimates of cost, using different frequencies of rainfall, were made and, as a result, in the computation of the volumes of storm-water to be handled by the Olentangy-Scioto and the South Side Intercepting Sewers, the rainfall intensity curves used were such that, on the average, the

combined sewers in the central part of the city, from the Olentangy River to West Mound Street, will not overflow more than four times per year to the Scioto River, and along the Olentangy River and along the Scioto below West Mound Street, not more than eight times per year.

Hydraulics of Sewers.—In determining the sizes of the new intercepting sewers (concrete with vitrified clay lining), Kutter's formula, $n = 0.013$, was used, whereas, in computing the capacities of existing sewers, n was taken as 0.015. All sewers were assumed to flow full. As the flow in the intercepting sewers is largely from combined sewers, slopes were adopted which, in general, give velocities of not less than 2.5 ft per sec for present average rates of dry-weather flow. In one stretch of the Olentangy-Scioto Intercepting Sewer a velocity of approximately 2.25 ft per sec was used, and, in another, where the sewer carries sanitary sewage only, a velocity of approximately 2 ft per sec was assumed. For estimated 1960 conditions, however, these velocities will be not less than 2.5 ft per sec.

The Olentangy-Scioto Intercepting Sewer

The Olentangy-Scioto Intercepting Sewer, as planned, extends from the Sewage Treatment Works to Orchard Lane (see Fig. 1). A profile is shown in Fig. 3. The estimated rates of sewage flow in this sewer in 1945 and in 1960 are given in Table 2(*a*).

TABLE 2.—Estimated Rates of Sewage Flow in 1945 and 1960.

Item No.	Location	Rates of Flow, in Million Gallons Daily									
		Average								Maximum assumed for design	
		1945				1960				1945	1960
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	(*a*) In the Olentangy-Scioto Intercepting Sewer										
1	Olentangy-Scioto Intercepting Sewer above the South Side Intercepting Sewer connection	30.0				36.6					
2	South Side Intercepting Sewer	4.2				5.6					
3	At Whittier Street Storm Stand-by Tanks (Item No. 1 plus Item No. 2)		34.2				42.2			68.4	84.4
4	Main Intercepting Sewer		6.4				10.9				
5	Alum Creek Intercepting Sewer		10.9				15.7				
6	Whittier Street Storm Stand-by Tanks to Main Sewage Pumping Station (sum of Items Nos. 3, 4, and 5)			51.5				68.8		103	138
7	West Side Sanitary and Old 4-Foot Dry Flow Sewers			8.1				11.2			
8	Main Sewage Pumping Station to Sewage Treatment Works (Item No. 6 plus Item No. 7)				59.6				80.0	120	160
	(*b*) In the Alum Creek Intercepting Sewer										
9	Alum Creek Sewer above Main Street	2.8				4.0					
10	Main Street Sewer	5.0				6.7					
11	At Alum Creek Storm Stand-by Tank (Item No. 9 plus Item No. 10)		7.8				10.7			15.0	22.3
12	Area east of Alum Creek *		2.5				4.0				
13	At Livingston Avenue (Item No. 11 plus Item No. 12)			10.3				14.7		21.7	29.8
14	Industrial Avenue Sewer and area south-east of the City and west of Alum Creek			0.6				1.0			
15	At Whittier Street Storm Stand-by Tanks (Item No. 13 plus Item No. 14)				10.9				15.7	22.6	31.5

* Includes Village of Hanford, west of Alum Creek.

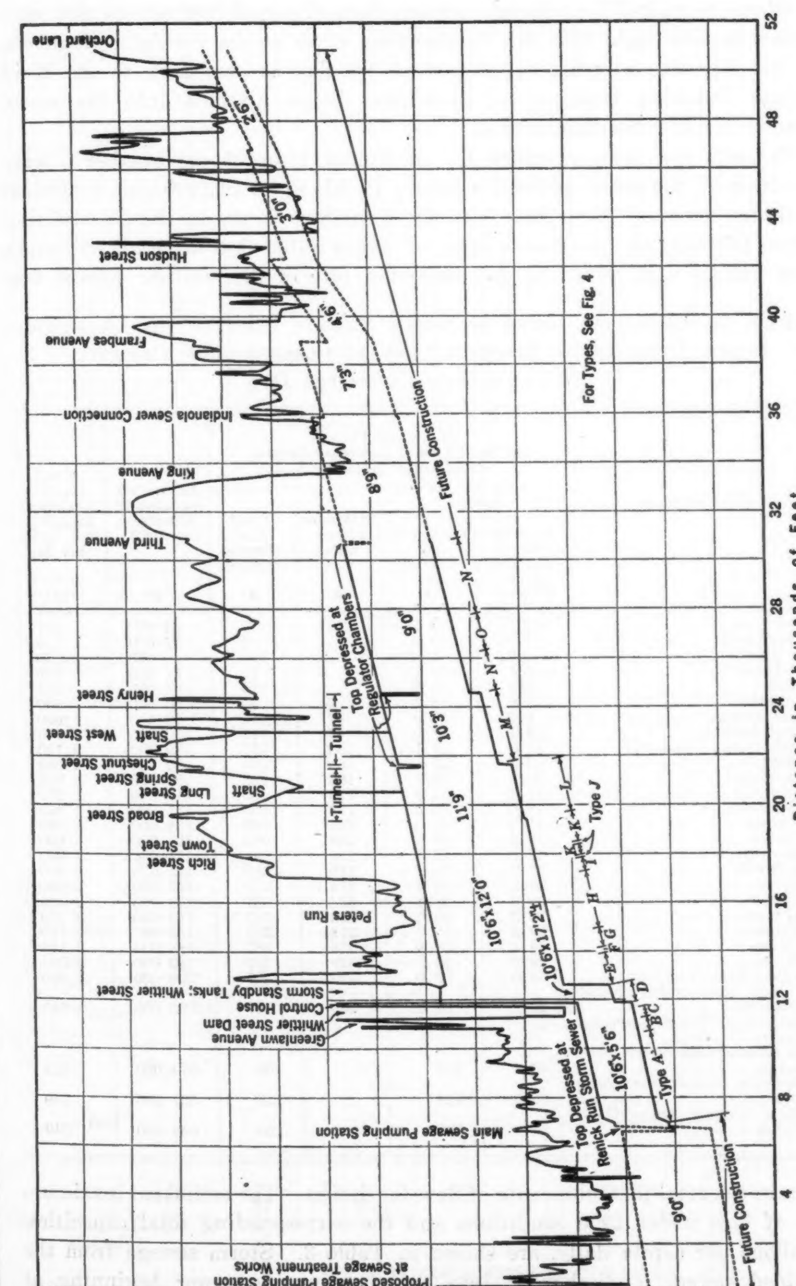

FIG. 3.—PROFILE OF OLENTANGY-SCIOTO INTERCEPTING SEWER, COLUMBUS, OHIO.

Storm Sewage Flow.—Storm sewage, from the combined sewers now connected, is discharged into this intercepting sewer at the regulator chambers. At the Whittier Street tanks a part of the flow is passed on to the Main Sewage Pumping Station, the remainder being diverted into the storm stand-by tanks for sedimentation.

Capacity has been provided for maximum rates of dry-weather sewage flow from all the areas assumed tributary to this sewer in 1960 plus controlled maximum rates of storm flow from the combined sewers in the Intercepting Sewer District, the maximum rate of storm-water flow at each collection point having been added to the maximum rate of dry-weather sewage flow

TABLE 3.—Estimated Rates of Storm Sewage Flow in the Olentangy-Scioto Intercepting Sewer in 1960 and Corresponding Capacity, in Gallons per Capita per Day.

Collection point	Rates of Flow Beginning at Collection Points, in Million Gallons Daily				Estimated population tributary in 1960	Total capacity provided, in gallons per capita per day
	Sewage		Maximum storm-water flow	Total flow (figures rounded)		
	Average dry-weather flow	Maximum dry-weather flow				
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Orchard Lane	3.5	9.1	...	9	35 000	257
Brighton Road	3.7	9.4	...	9	37 000	243
Weber Road	4.0	9.8	...	10	40 000	250
Sunset Drive	5.1	12.2	...	12	51 000	235
Glen Echo	5.4	12.6	...	13	54 000	241
Hudson Street	5.9	13.5	20	34	56 500	600
Frambes Avenue	8.2	18.0	97	115	69 400	1 660
Woodruff Avenue	8.4	18.0	97	115	70 000	1 640
Indianola Sewer Connection	11.9	24.5	162	187	89 000	2 100
King Avenue	12.5	25.5	162	188	92 100	2 040
Fourth Avenue	12.6	26.0	162	188	92 800	2 030
Third Avenue	15.8	31.8	173	205	110 000	1 870
Second Avenue	16.0	32.0	173	205	111 000	1 850
Henry Street	21.0	42.0	226	268	138 000	1 940
West Street	23.3	46.6	238	285	151 000	1 890
Chestnut Street	27.7	55.4	289	344	174 000	1 980
Spring Street	27.9	55.8	310	366	175 000	2 090
Long Street	28.2	56.4	314	370	177 000	2 090
Broad Street	29.3	58.6	328	387	183 000	2 110
Capital Street	29.3	58.6	328	387	183 000	2 110
State Street	29.3	58.6	328	387	183 000	2 110
Town Street	29.4	58.8	328	387	183 000	2 110
Rich Street	29.6	59.2	328	387	185 000	2 090
Peters Run Sewer Connection	34.0	68.0	339	407	208 000	1 960
South Side Intercepting Sewer Connection	42.2	84.4	403	487	265 000	1 840
Whittier Street Storm Stand-by Tanks:						
Main Intercepting Sewer Connection	53.1	106	...	106	374 000	284
Alum Creek Intercepting Sewer Connection	68.8	138	...	138	531 000	259
West Side Sanitary Sewer Connection	80.0	160	...	160	643 000	249

to obtain the total maximum rate of flow for design. The estimated maximum rates of flow under 1960 conditions and the corresponding total capacities, in gallons per capita daily, are shown in Table 3. Storm sewage from the combined sewers is discharged into this intercepting sewer beginning at Hudson Street, and from this point to the Whittier Street tanks. In this

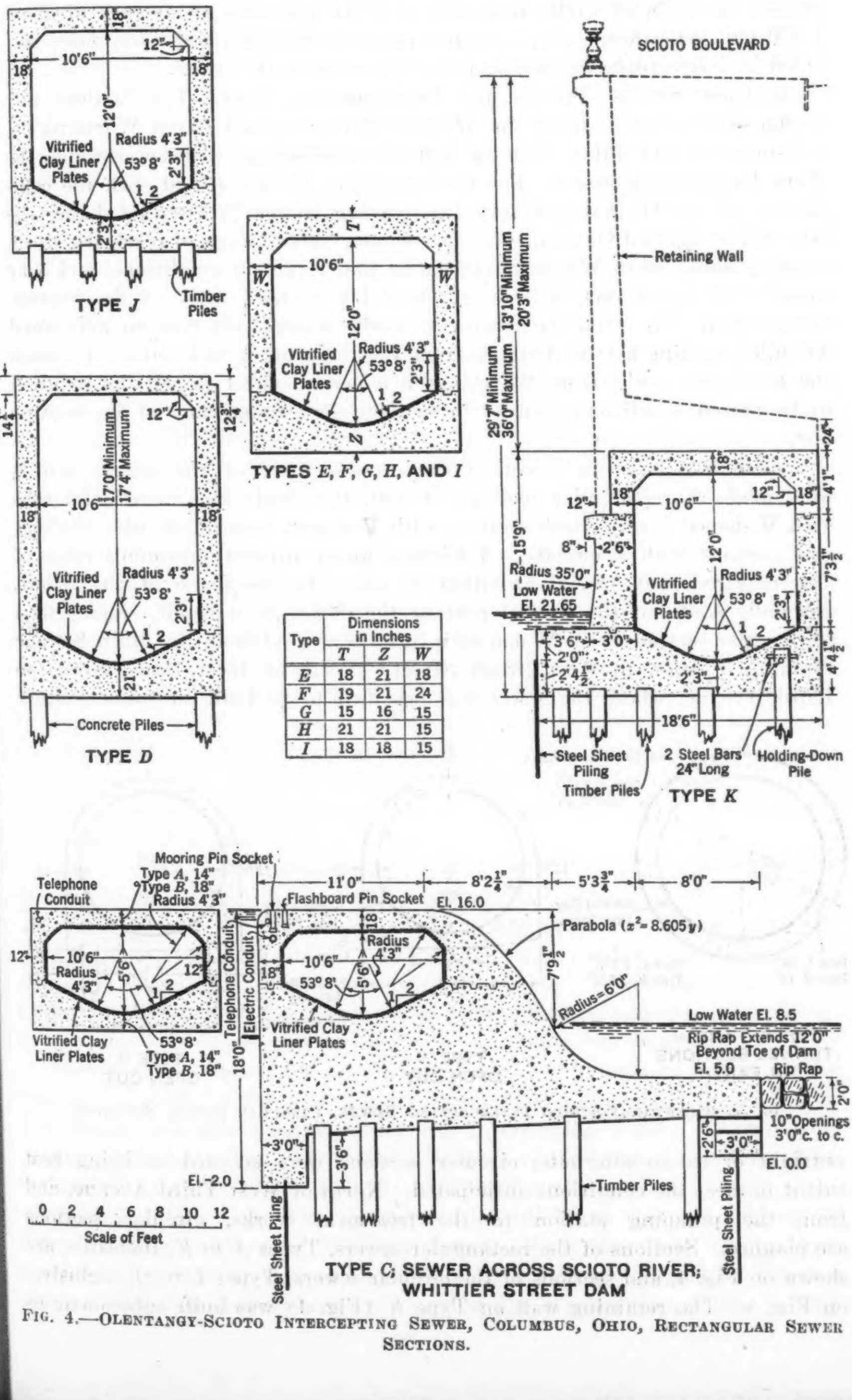

FIG. 4.—OLENTANGY-SCIOTO INTERCEPTING SEWER, COLUMBUS, OHIO, RECTANGULAR SEWER SECTIONS.

section the capacity varies from 600 to 2 110 gal, with an average of about 1 900 gal, per capita daily, a greater capacity than has heretofore been provided in intercepting sewers, as far as the writers are aware.

Distance Sewage Travels and Corresponding Time.—The farthest any sewage will travel to reach the Whittier Street tanks is from Worthington, a distance of 10.1 miles. For the first 2.9 miles sewage will flow through the Main Intercepting Sewer. For the remaining 7.2 miles, and with the completion of the Olentangy-Scioto Intercepting Sewer to Orchard Lane, the flow will be divided between these two sewers. For sewage to reach the storm stand-by tanks from Worthington, 6.8 hr under present conditions, and 5 hr under 1960 conditions, will be required for average rates of dry-weather sewage flow. To reach the treatment works sewage will flow an additional 2.6 miles, making a total distance from Worthington of 12.7 miles. To reach the treatment works from Worthington a period of 8.1 hr will be required under present conditions, and 6.1 hr in 1960, for average rates of dry-weather flow.

Sewer Sections.—In selecting the cross-sections of the sewer, studies were made of rectangular sections of both the single and twin-barrel type with **V**-shaped inverts, arch sections with **V**-shaped inverts, circular sections, and sections with a cunette. Velocities under present minimum rates of flow required that special attention be given to the shape of the invert, especially south of the central part of the city. As a result, single-barrel rectangular sections with **V**-shaped inverts were used from the Main Sewage Pumping Station to West Broad Street. North of West Broad Street to Third Avenue, where the sewer was specified to be built in tunnel, either

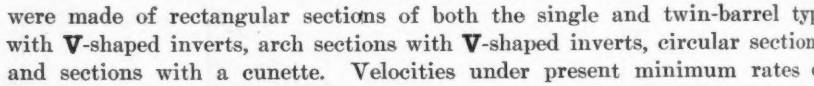

FIG. 5.—OLENTANGY-SCIOTO INTERCEPTING SEWER, CIRCULAR SEWER SECTIONS.

outright or as an alternate, circular sections were adopted as being best suited to meet the conditions anticipated. North of West Third Avenue, and from the pumping station to the treatment works, circular sections are planned. Sections of the rectangular sewers, Types *A* to *K*, inclusive, are shown on Fig. 4, and sections of the circular sewers, Types *L* to *O*, inclusive, on Fig. 5. The retaining wall on Type *K* (Fig. 4) was built subsequent to

the construction of the sewer, which was designed structurally and architecturally to support the wall.

Types A and B.—Although the interior dimensions of Types *A* and *B* are the same, it was necessary to vary the exterior dimensions and the amount of steel reinforcement to meet different conditions of loading. Both types were designed to resist an internal pressure of about 33 ft of water above the invert, a condition which might occur should the pumping station be shut down temporarily. Back-filling was not permitted until the concrete in the top slab was at least 21 days old.

Type C.—Whittier Street Dam.—At the Scioto River the Type *C* sewer (Fig. 4), was extended across the river within a low overflow dam, approximately 520 ft. long, to avoid the use of an inverted siphon. At the sub-grade of the dam the river bed is pervious gravel. Experience at Columbus, with bridges and other structures founded on such material, has indicated the necessity of pile foundations and precautions to prevent under-cutting during flood-stages. The structure, therefore, was supported on piles and was protected against under-cutting by cut-off walls of steel sheet-piling. At both ends of the dam, the lines of sheet-piling were connected, thus forming a complete cut-off.

The structure is stable, when the sewer is either empty or full: (*a*) With the pool above the dam empty; (*b*) with the pool full to the top of the flash-boards at Elevation 20.0; (*c*) with the pool full to the top of the flash-boards and with upward pressure; and (*d*) with the pool overflowing, with the water surface at Elevation 38.0. Under each of these conditions the resultant force strikes the base within the middle-third. The most severe combination of conditions is with the sewer empty, and with the pool full to the top of the flash-boards, and with upward pressure, when the factor of safety against overturning is 5.2. Upward pressure was assumed as full-head pressure at the heel, decreasing uniformly to zero at the toe, and acting on two-thirds of the area of the base.

The dam was constructed in 30-ft sections within an earth coffer-dam. Driving of the sheet-piling cut-offs was continued until, with the equipment and method used, further penetration was not possible. The penetrations obtained, however, were such as to indicate that the piling extended into blue clay, shown by borings to underlie the gravel. The bearing value of the piles was computed by the *Engineering News* formula, and they were driven until a safe bearing value of at least 20 tons per pile was obtained.

Provision has been made at the north end for by-passing a part of the flow of the river, or for lowering the water level in the pool, through four 4-ft square passages passing under the intercepting sewer. Double rows of stop-planks have been provided at both ends of the by-pass structure which is founded on 14-in. square pre-cast reinforced concrete piles.

Type D.—This type (Fig. 4) was constructed along the west side of the storm stand-by tanks near Whittier Street. With the sewer empty, with the levee immediately west of it in place, and with no support on the east side (a condition possible not only during the construction of the present

installation, but also when additional tanks are built), the structure would not be in equilibrium against side-thrust. Therefore, two rows of pre-cast reinforced concrete piles were driven under the sewer to which the structure was anchored. These piles also serve to hold the structure down when hydrostatic uplift is developed with the sewer empty and with high water in the river. Leading to each storm stand-by tank are four inlet openings, 4 ft wide by 4 ft 2 in. high, the bottoms of which are approximately 7 ft 3 in. above the invert of the sewer. There is, also, an outlet opening, with its bottom about 2 ft 3 in. above the invert of the sewer, through which material deposited in the tank is flushed into the sewer. Openings, with bulkheads, have also been provided for two additional tanks.

Types E, F, G, H, I, and J.—Each of these types (Fig 4) was designed not only to support external loads but also to resist an internal pressure of about 17 ft of water above the invert, which would obtain with the gates of the storm stand-by tanks shut off and with the regulator and emergency outlet gates in the Control House closed. Back-filling was not permitted until the concrete in the top slab was 21 days old.

Type K.—Type *K* (Fig. 4) was designed to support a retaining wall, with back-filling, and is supported on piles. To resist overturning, the structure was anchored to each pile in the row under the heel. Protection against erosion was provided by driving a steel sheet-pile cut-off wall along the river face of the concrete base. The structure was constructed within an earth coffer-dam.

Types L and M.—Under the railroad tracks north of Chestnut Street, Type *M* was built of reinforced concrete; otherwise, Types *L* and *M* (Fig. 5) were constructed of plain concrete. Each type was built through glacial drift, in tunnel, under compressed air. The air pressures used varied from 3 to 8 lb per sq in., and voids behind the tunnel lining were filled by pressure grouting. On Type *L*, the grout averaged 0.18 cu yd, and on Type *M*, 0.20 cu yd, per ft of sewer.

Types N and O.—These types (Fig. 5) were built in open trenches, Type *N* where the sub-soil provided sufficient bearing power, and Type *O* where the bearing power was insufficient.

South Side Intercepting Sewer

The South Side Intercepting Sewer will connect with the Olentangy-Scioto Intercepting Sewer at West Whittier Street, just above the storm stand-by tanks (Fig. 1). It varies in diameter from 63 to 72 in. and is for the purpose of conveying sewage from one district served by separate sewers and from two districts served by combined sewers. Capacity is provided for maximum rates of dry-weather sewage flow from the area assumed to be tributary in 1960 plus controlled maximum rates of storm-sewage flow from the two combined sewer districts, the maximum rate of storm-water flow at each collection point having been added to the maximum rate of dry-weather sewage flow to obtain the total maximum rate of flow for design.

Regulator Chambers

The dry-weather sewage flow from each combined sewer connected to one of the new intercepting sewers passes through a regulator chamber. During storms, however, the rate of flow of storm sewage is limited by the setting of a sluice-gate on the connection leading to the intercepting sewer. The storm sewage, in excess of that which passes into the intercepting sewer, accumulates and rises in the regulator chamber until it passes over the overflow weirs and thence through a channel leading directly to the river, or back to the combined sewer below the regulator chamber, which piece of sewer serves as a storm over-flow. The sluice-gate serves not only to limit the flow to the intercepting sewer but also to shut off the flow should that be desired. With the connection closed, sewage, in dry weather, or storm sewage, during storms, can pass over the overflow weirs. Under normal operating conditions the sluice-gate is partly open and set to limit the flow to that desired. The regulator chambers were designed to limit the number of storm over-flows to not more than 4 per year, on the average, in the central part of the city, where combined sewer outlets to the rivers are relatively close together, and not more than 8 per year, on the average, north and south of the city, where outlets are more widely scattered. Details of the Broad Street Regulator Chamber are shown in Fig. 6. Aside from the snow manhole, this regulator chamber is typical of those built at other locations.

Each regulator chamber is provided with electric lights set in water-tight marine fixtures and is also provided with hose connections for flushing. Stored in grooves below the floor are stop-planks so that the sluice-gate may be isolated for inspection and repairs. Two lengths of stop-planks were used, and a portable stop-plank lifter for each size has been provided.

To reduce the use of exposed iron work to a minimum, concrete balustrades were adopted instead of pipe railings. At the Chestnut Street Regulator Chamber an opening was provided through which a boat may be lowered into the intercepting sewer.

Alum Creek Intercepting Sewer

The Alum Creek Intercepting Sewer (Fig. 1) was placed in service on March 15, 1932. A profile, and sections of the sewer, Types *P* to *S*, inclusive, are shown, respectively, on Figs. 7 and 8. Sewage from the East Side District flows in this sewer to and into the Olentangy-Scioto Intercepting Sewer just below the storm stand-by tanks near Whittier Street. During and following storms a controlled volume of storm sewage is discharged into the sewer at the storm stand-by tank on Alum Creek. The estimated rates of sewage flow in 1945 and in 1960 are given in Table 2(*b*). The capacity of the sewer, however, exceeds the 1960 requirements because it was constructed by tunneling, and construction requirements rather than capacity needs governed the size. Where not built by tunneling the sewer was made proportionally larger, to conform to the capacity provided in the tunnel. From East Livingston Avenue to the storm stand-by tanks it was constructed of concrete, and north of East Livingston Avenue, of vitrified clay segment block.

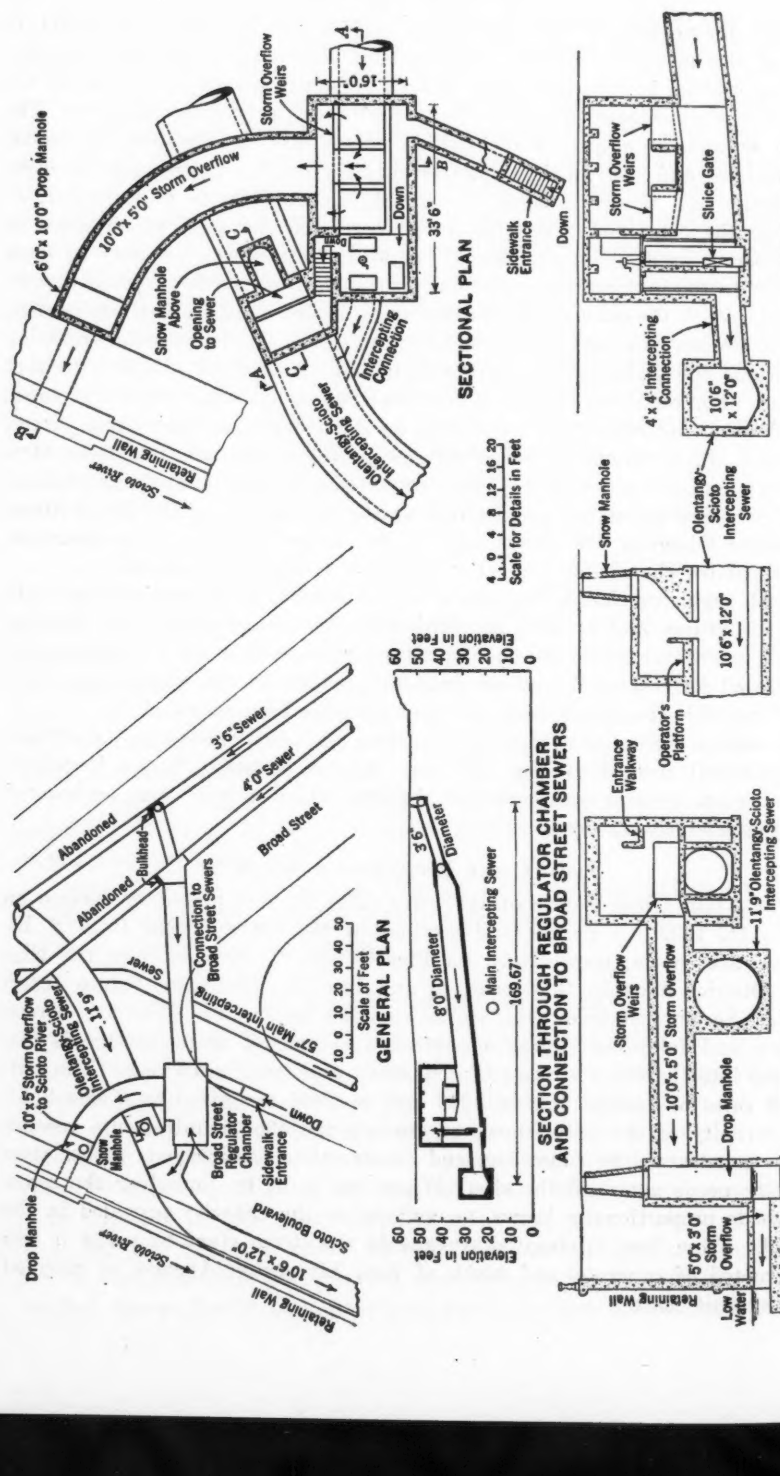

FIG. 6.—DETAILS OF BROAD STREET REGULATOR CHAMBER, COLUMBUS, OHIO.

Types P and Q.—These types (Fig. 8) were built by tunneling under compressed air. Under the railroad, and in a few cases, where unusually heavy roof loads were encountered, Type *P* was built of reinforced concrete.

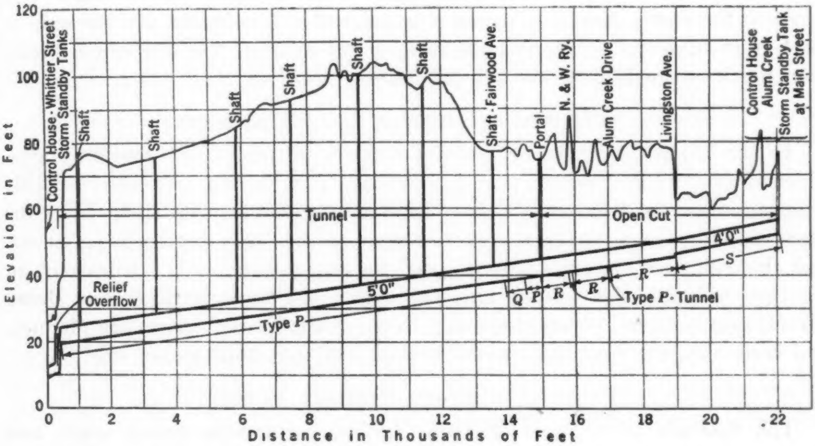

FIG. 7.—PROFILE OF ALUM CREEK INTERCEPTING SEWER, COLUMBUS, OHIO.

Elsewhere, plain concrete was used. Most of the tunnels were constructed from seven shafts,[9] 1 700 to 2 400 ft apart. The headings varied from 644 to 1 380 ft in length, the average being 1 039 ft. Air pressures varied from 6 to 35 lb, the average being about 15 lb per sq in. Voids behind the tun-

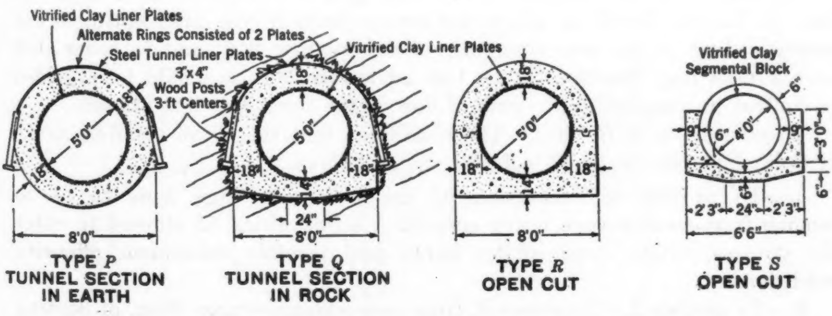

FIG. 8.—ALUM CREEK INTERCEPTING SEWER, CIRCULAR SEWER SECTIONS.

nel lining were filled by pressure grouting, the grout averaging 0.14 cu yd per ft of sewer.

Types R and S.—Although the average depth of the sub-grade of Type *R* (Fig. 8) was approximately 35 ft, the sewer was built in open trench. Type *S* (Fig. 8) was also built in open trench.

[9] "Surveying Under Compressed Air—Great Accuracy Obtained in Holing-Through Sewer Tunnel Headings at Columbus, Ohio," by Orris Bonney, M. Am. Soc. C. E., *Civil Engineering*, Vol. 3, (January, 1933), p. 31.

Alum Creek Sewer

The Alum Creek Sewer is shown in Fig. 1 extending from East Main Street to the north corporation line to collect sewage from two combined sewer districts which are to be converted to the separate system, and from areas to the north, now only sparsely populated. As planned, the sewer is to be 4 ft in diameter. The average rate of dry-weather flow for which provision has been made is 4 000 000 gal daily.

Conversion of Combined Sewer Districts

In the Intercepting Sewer District a few small areas are served by combined sewers. Elevations of these sewers are such as not to permit of their connection to the Olentangy-Scioto Intercepting Sewer. In these areas the plans call for new sanitary sewers, tributary to the Main Intercepting Sewer, and the combined sewers are to be used for storm-water only. Similarly, in the East Side District, to eliminate two storm over-flows north of East Broad Street, new sanitary sewers, tributary to the Alum Creek Sewer, are planned, and here, also, the combined sewers will be used for storm-water only.

Function of Storm Stand-by Tanks

The function of a storm stand-by tank is to remove heavy solids and floating materials from excess storm-sewage flows prior to their discharge into a body of water. Furthermore, with storms giving a volume of flow less than that needed to fill the tank and to cause over-flow, a storm stand-by tank acts as a detention basin, storing excess storm sewage at peak flows and returning it to the sewer after the storm is over. Ordinarily, the tank would be located adjacent to an intercepting sewer and at the point where excess storm flows would be diverted. Unless the tank is to be unwatered by pumping, its bottom should be above the sewage level in the intercepting sewer corresponding to the maximum rate of dry-weather flow, and in order that such a tank may function to the best advantage, means should be provided to control, automatically, the rate of flow to the sewer below the tank.

Requirements to Be Met.—The conditions that the storm stand-by tanks at Columbus were designed to fulfill are, as follows:

1.—To provide sedimentation of excess storm-sewage flows so as to remove from such sewage heavy suspended solids which, if allowed to enter the streams, might cause sludge banks and possible subsequent offensive conditions.

2.—To provide for the removal, from excess storm-sewage flows, of floating solids which might cause unsightly conditions if allowed to enter the streams.

3.—To drain the stored sewage back into the intercepting sewers after a storm is over and after the flow in the intercepting sewers has decreased to less than twice the average rate of dry-weather flow.

4.—To be capable of rapid and easy cleaning and to be arranged so that solids deposited in the tanks can be flushed back into the intercepting sewers and carried to the treatment works.

5.—To be capable of handling the entire storm flow in the intercepting sewers in case the treatment works should be temporarily out of service.

The conditions which the control works were designed to fulfill are:

1.—To provide automatic control of the volume of storm sewage flowing to the treatment works so that, at no time, will the capacity of the works be exceeded.

2.—At the Whittier Street tanks, to provide automatic control which will close off the tanks from the Scioto River during periods of extreme high water, at which time the tanks would otherwise be flooded.

Basis of Design.—As far as the writers are aware the storm stand-by tanks at Columbus were the first to be placed under construction in the United States. Owing to the absence of data on the operation of such tanks in this country, it was necessary to make certain assumptions. Those made, were

TABLE 4.—PERIODS OF DETENTION IN THE STORM STAND-BY TANKS FOR DIFFERENT RATES OF SEWAGE FLOW.

Flow in sewer, in million gallons daily (1)	Ratio of storm flow to average dry-weather flow (2)	Storm flow passing tanks to sewage treatment works, in million gallons daily (3)	EXCESS STORM FLOW PASSING THROUGH TANKS: Excess storm flow, in million gallons daily (4)	Rates of excess storm flow to average dry-weather flow (5)	Period of detention in storm stand-by tanks, in minutes (6)
(a) OLENTANGY-SCIOTO INTERCEPTING SEWER AT WHITTIER STREET. TANKS UNDER ESTIMATED 1945 CONDITIONS†					
34	1	34	0	0	....
68	2	34	0	0	....
102	3	68	34	1	180
136	4	68	68	2	90
170	5	68	102	3	60
204	6	68	136	4	45
238	7	68	170	5	36
272	8	68	204	6	30
306	9	68	238	7	25.7
340	10	68	272	8	22.5
374	11	68	306	9	20
408	12	68	340	10	18
442	13	68	374	11	16.4
487*	14.3	68	419	12.3	14.6
487*	14.3	0	487	14.3	12.6
(b) MAIN STREET SEWER AT ALUM CREEK TANK. UNDER ESTIMATED 1960 CONDITIONS					
6.7	1	6.7	0	0	....
13.4	2	13.4	0	0	....
20.1	3	13.4	6.7	1	180
26.8	4	13.4	13.4	2	90
33.5	5	13.4	20.1	3	60
40.2	6	13.4	26.8	4	45
46.9	7	13.4	33.5	5	36
53.6	8	13.4	40.2	6	30
60.3	9	13.4	46.9	7	25.7
67.0	10	13.4	53.6	8	22.5
73.7	11	13.4	60.3	9	20
80.4	12	13.4	67.0	10	18
87.1	13	13.4	73.7	11	16.4
93.8	14	13.4	80.4	12	15
100	15	13.4	87.1	13	13.8
134	20	13.4	121	18	10
201	30	13.4	188	28	6.4
268	40	13.4	255	38	4.7
335	50	13.4	322	48	3.8
388 *	58	13.4	375	56	3.2
388 *	58	0	388	58	3.1

* Maximum rates of flow in sewers. † Three tanks in service.

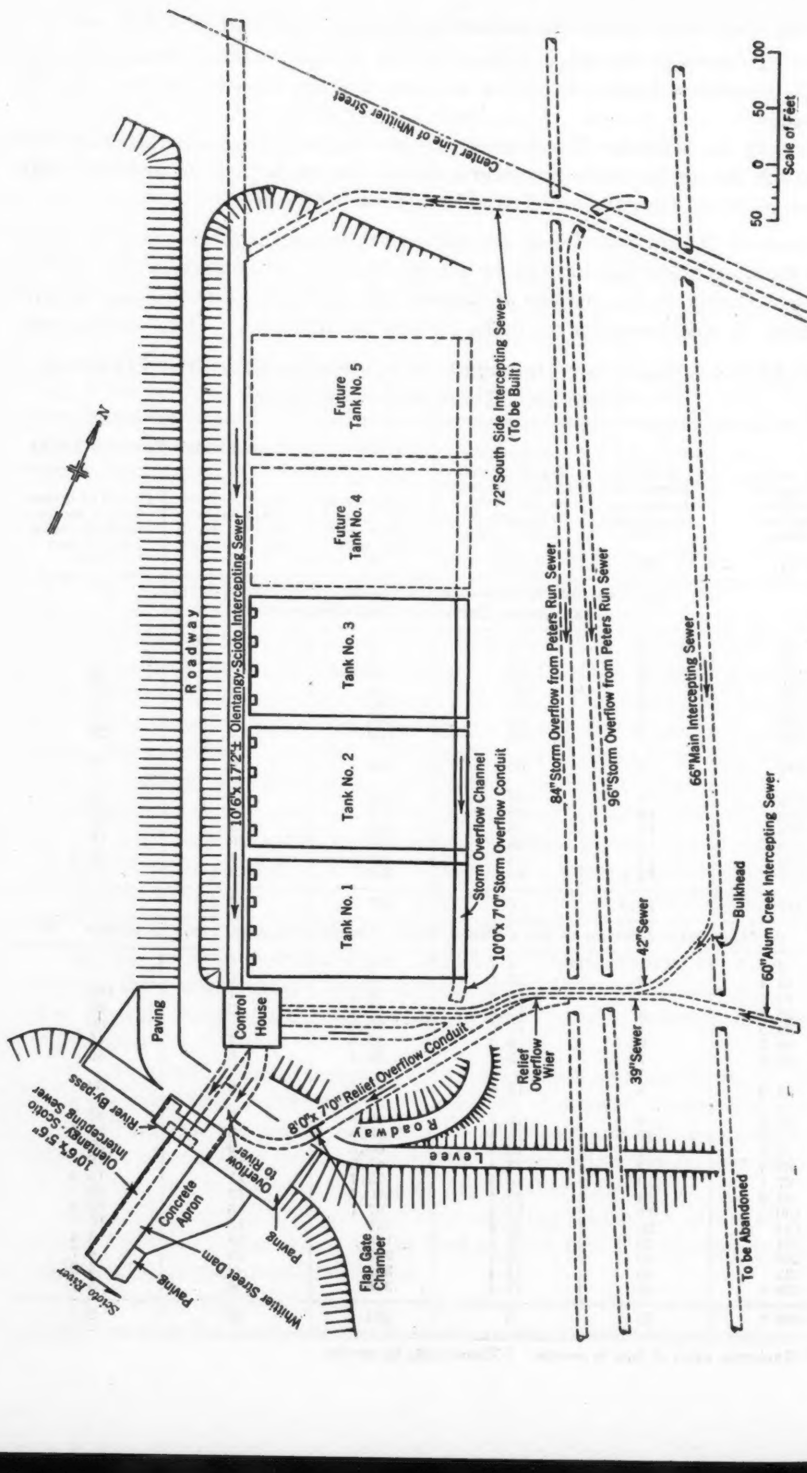

FIG. 9.—PLAN OF STORM STAND-BY TANKS, ON WHITTIER STREET, COLUMBUS, OHIO.

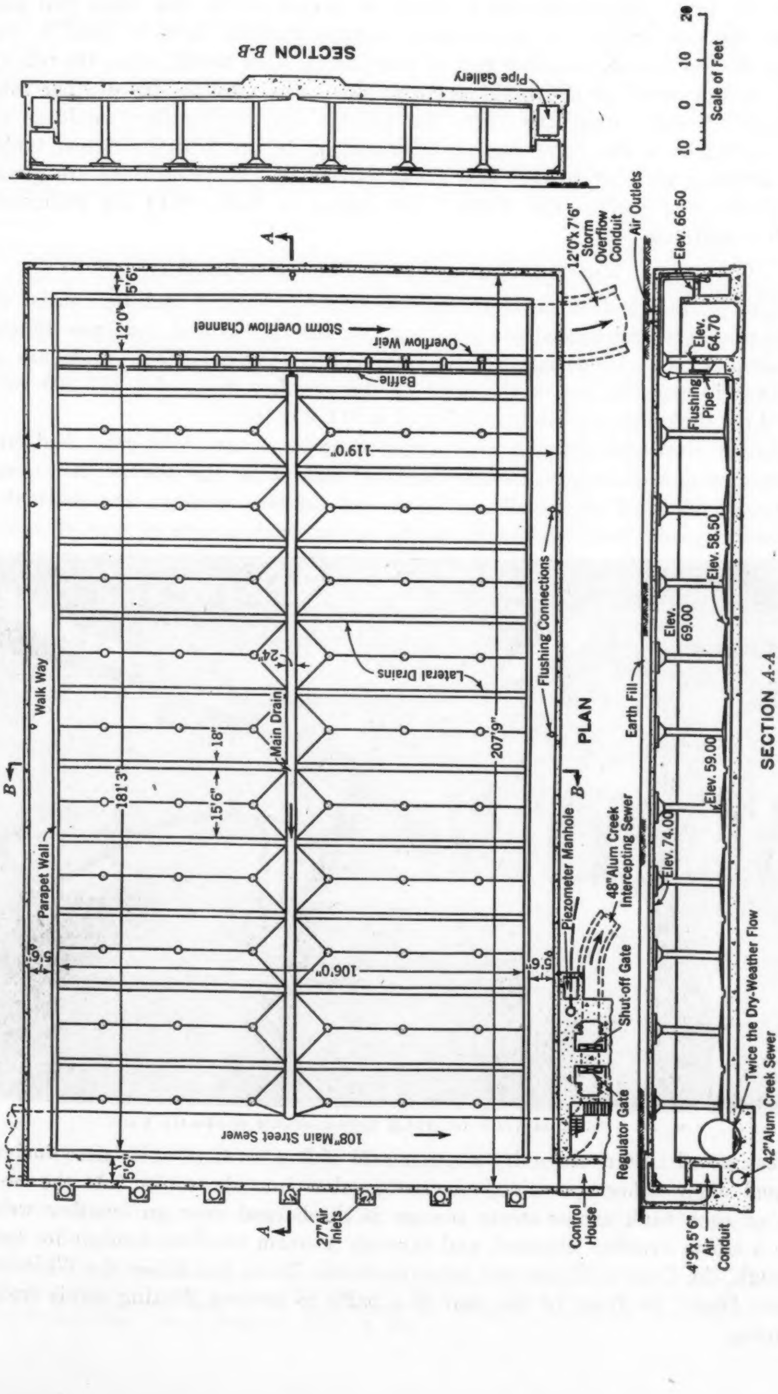

FIG. 10.—DETAILS OF ONE OF THE STORM STAND-BY TANKS ON WHITTIER STREET, COLUMBUS, OHIO.

that the tanks should provide a detention period of 30 min when full and when the rate of flow of excess storm sewage through them is equal to six times the average dry-weather rate of flow; or, in other words, when the rate of flow in the sewer at the tanks is eight times the average dry-weather rate of flow, of which total flow twice the average dry-weather flow would go to the treatment works. The periods of detention in the Whittier Street tanks for different rates of sewage flow under estimated 1945 conditions are given in Table 4(*a*) and in the Alum Creek tanks in Table 4(*b*) for estimated 1960 conditions.

Whittier Street Storm Stand-By Tanks

The storm stand-by tanks at Whittier Street, Figs. 9 and 10, consist of three uncovered reinforced concrete tanks on the east side of and immediately adjacent to the Olentangy-Scioto Intercepting Sewer. Each tank has a volumetric capacity, below the crest of the overflow weir, of 1 337 000 gal; the three tanks have a total capacity of 4 011 000 gal.

In the west wall of each tank are four 48-in. square inlet-gates and one 24-in. circular drain-gate, connecting the tank with the sewer. All these gates are operated electrically. The inverts of the openings are above the hydraulic grade that will obtain in the sewer, with a rate of flow of twice

Fig. 11.—Interior View of Alum Creek Storm Stand-By Tank.

the estimated 1960 average dry-weather rate of flow, so that, at no time during normal dry-weather operation, can sewage flow into the tanks. At the east end of each tank excess storm sewage is discharged over an overflow weir into a storm overflow channel, and through a storm overflow conduit to, and through, the Control House and into the Scioto River just below the Whittier Street Dam. In front of the weir is a baffle to prevent floating solids from escaping.

A movable tank cleaner (Fig. 11) is fitted to the coping of each longitudinal wall of each tank. It consists of a standard piece of fire-fighting equipment known as a "cellar pipe," mounted on a movable carriage which runs on rails set in the wall copings. This cleaner is used to flush the deposited matters back into the intercepting sewer after the tanks have been drained. At one corner of each tank is a manhole containing an electrical pressure-switch which makes an electrical contact when the level of the storm sewage in the tank reaches the elevation of the crest of the overflow weir. With high water in the river the empty tanks are subjected to upward water pressure, in excess of their weight, and it was necessary, therefore, to provide means for preventing flotation. This was done by driving pre-cast reinforced concrete holding-down piles[10] to which the tanks were anchored by extending the longitudinal reinforcement bars of the piles into the floors.

Whittier Street Control Works

When the Olentangy-Scioto Intercepting Sewer is completed, when all the lateral sewers are connected to it, and when certain small areas are converted from the combined to the separate system, the Main Intercepting Sewer will become a sanitary sewer discharging into the Olentangy-Scioto Intercepting Sewer below the Whittier Street tanks. The Alum Creek Intercepting Sewer also discharges into the Olentangy-Scioto Intercepting Sewer below the storm stand-by tanks near Whittier Street, the quantity of storm sewage entering the Alum Creek Intercepting Sewer being regulated at the Alum Creek Storm Stand-By Tank. Consequently, the rate at which storm sewage from the Olentangy-Scioto Intercepting Sewer can flow past the Whittier Street tanks and on to the treatment works will vary continuously, since it is the difference between the maximum rate of flow which the treatment works can handle and the combined rate of flow in the Main and Alum Creek Intercepting Sewers. The rate of flow in the Olentangy-Scioto Intercepting Sewer below the storm stand-by tanks near Whittier Street is regulated by locating the junction of the Main and Alum Creek Intercepting Sewers with the Olentangy-Scioto Intercepting Sewer in the Whittier Street Control House below the regulator gates on this sewer and then by controlling, automatically, the action of the regulator gates from the sewage level in this sewer below the junction.

The control works, in the Control House (Figs. 12 and 13), consist of electrically operated sluice-gates, two duplicate electric float-switches, one regulator float-switch, and a control panel, together with auxiliary electrical equipment.

Shut-Off Gates.—Three gates, 48 by 72 in., permit the shutting off of the Olentangy-Scioto Intercepting Sewer so that no sewage from this sewer or from the Main and Alum Creek Intercepting Sewers, can reach the treatment works. These gates are normally open, being closed only if the pumping station is shut down.

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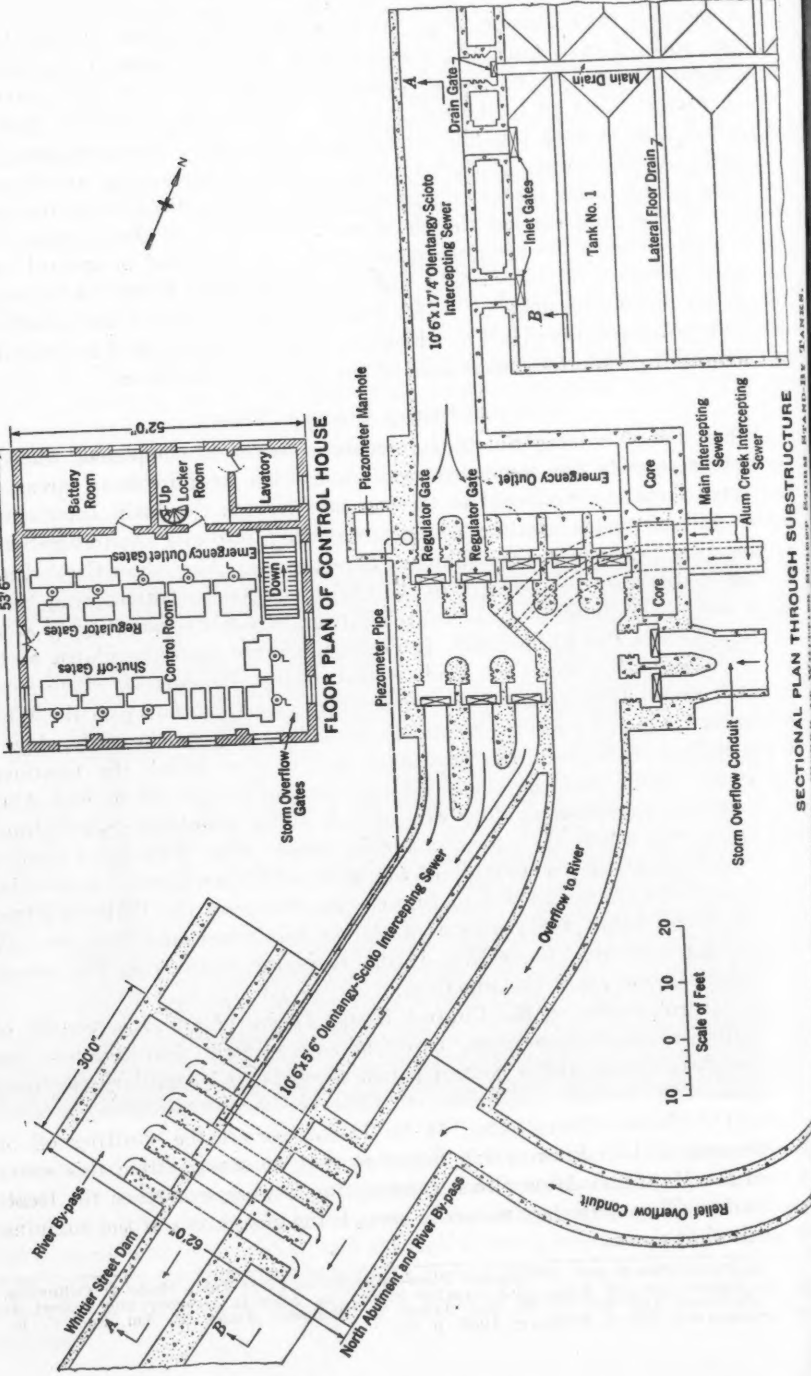
[10] "Holding-Down Power of Concrete Piles—Results of Field Tests Made at Columbus, Ohio, on Square Pre-cast Piles with Parallel Sides," by John H. Gregory and Robert A. Allton, Members, Am. Soc. C. E., and James H. Blodgett, Assoc. M. Am. Soc. C. E., *Civil Engineering*, Vol. 3, February 1933, p. 66.



SECTIONAL PLAN THROUGH SUBSTRUCTURE

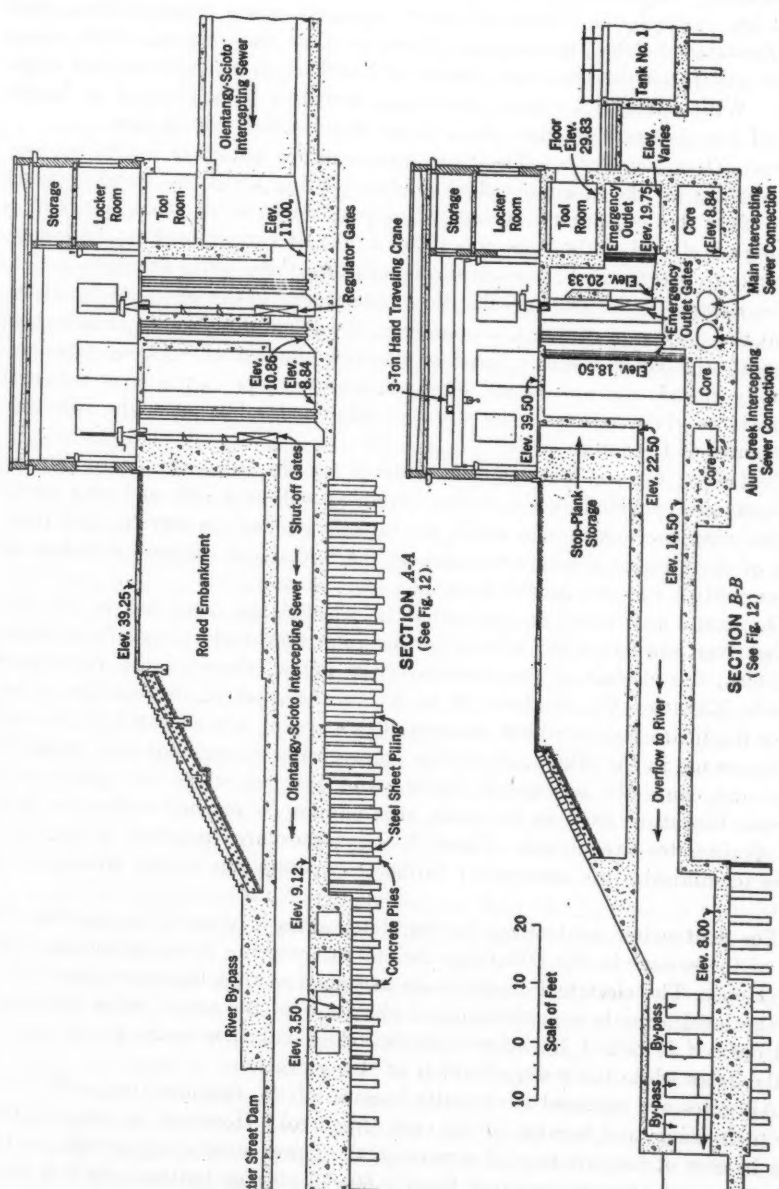


FIG. 13.—SECTIONS OF CONTROL WORKS AT WHITTIER STREET STORM STAND-BY TANKS.

Regulator Gates.—By means of float control, the two regulator gates, 42 by 72 in., control the volume of storm sewage passing through them from the Olentangy-Scioto Intercepting Sewer so that the rate at which storm sewage will reach the treatment works will be held to a predetermined maximum. With additional sewage treatment facilities it is planned to handle rates of flow as great as twice the average dry-weather rate of flow.

Storm-Overflow Gates.—The two storm-overflow gates, 48 by 84 in., permit shutting off the storm-overflow conduit leading from the stand-by tanks to the river so that, during periods of extreme high water when the river is at a higher elevation than the top of the tanks, it cannot back up and cause the tanks to overflow. Under normal conditions, these gates are open.

Emergency Outlet Gates.—The three emergency outlet gates, 48 by 84 in., permit the discharge of storm sewage from the Olentangy-Scioto Intercepting Sewer to the river without passing through the tanks. These gates are normally closed, and are opened only when high water in the river makes it necessary to close, not only the storm-overflow gates but, also, the inlet and drain-gates of the tanks.

Control of Gates.—Push-button controls for the inlet and drain-gates of the tanks are provided on the control panel as well as a jack and plug device for changing the sequence in which the tanks are placed in service. The terminals of the electric pressure switches at the tanks, and solenoid switches for proper control, are also provided on the control board.

Duplicate float-switches, connected in parallel, are actuated by the rise of the river and cause the closing of the inlet and drain-gates of the tanks, and, also, the closing of the storm-overflow gates, whenever the river stage exceeds Elevation 25, or about 15 in. below the crest of the overflow weirs. Other duplicate float-switches, connected in parallel, are actuated by the rise of the sewage in the Olentangy-Scioto Intercepting Sewer above the regulator gates and cause the emergency outlet gates to open when the sewage level exceeds Elevation 29.5, an elevation that cannot be reached unless the inlet and drain-gates are closed. These float-switches are installed in duplicate so as to minimize the chances of failure to operate due to the sticking of a float.

The float-switch controlling the regulator gates is actuated by the rise and fall of the sewage in the Olentangy-Scioto Intercepting Sewer below the Control House. The electrical contacts are arranged so that the gates close if the sewage level exceeds a predetermined elevation in the sewer below the gates and open if it falls 1 in. below this elevation; but they cause no movement of the gates while the sewage level is at this elevation.

All gates are operated electrically because of the freedom from difficulties due to freezing, and because of the ease of control. However, to insure operation in case of interruption of service in the power supply, all motors are for direct current and are operated from a 60-cell storage battery which is kept charged automatically by a motor-generator set.

Battery Room.—In the battery room are the switchboard, transformers, motor-generator set, and storage battery. The motor-generator set is connected in parallel with the storage battery and has a capacity of 14 amperes

with a full load of 140 volts, sufficient to operate the two regulator gates without drawing on the battery. A contact on the regulator float-switch starts the motor-generator set before the regulator gates go into operation. The storage battery is of sufficient capacity to deliver 40 amperes continuously for 8 hours, with a final pressure of not less than 105 volts.

Stop-Planks.—In front of all sluice-gates are stop-plank grooves so that any gate may be isolated for repair or adjustment. The stop-planks and a stop-plank lifter are stored in grooves below the floor of the control room.

Operation During Dry Weather.—When the Olentangy-Scioto Intercepting Sewer is carrying only the dry-weather sewage flow and the river is at or below Elevation 25, the emergency outlet gates are closed, whereas the shut-off, regulator, and storm-overflow gates are open. The inlet gates and drain-gate of Tank No. 1 are open, whereas the gates on the other two tanks are closed. All the sewage in the intercepting sewer passes the tanks and flows through the regulator and shut-off gates to the pumping station and treatment works.

Operation During Storms.—Whenever a storm occurs, and the flow of sewage in the Olentangy-Scioto Intercepting Sewer increases, the level in the sewer below the Control House rises, thereby raising the float of the regulator float-switch. When the flow to the treatment works has reached the predetermined maximum rate and the level below the regulator gates is at the corresponding elevation, a contact is made in the regulator float-switch. The regulator gates then begin to close and continue to close until they throttle the flow sufficiently to cause the level in the sewer below the gates to fall, whereupon the contact breaks and the gates come to rest. If the level in the sewer below the gates now falls 1 in. below the predetermined elevation, a second contact is made in the regulator float-switch and the gates start to open, thus permitting more sewage to pass the gates and the sewage level below them to rise, until this second contact breaks and the gates again come to rest. From this time, as long as the storm continues, the gates are constantly "hunting," and maintain a level in the sewer below the gates as close to the desired elevation as possible. After the storm is over, and the flow in the intercepting sewer becomes less than the volume to be delivered to the treatment works, the second contact is made and the gates return to the open position.

As soon as the regulator gates come into action, the storm sewage in the intercepting sewer above these gates begins to back up and flow into Tank No. 1, the gates of which are open. When this tank has filled to the elevation of the overflow weir, a contact is made in the pressure-switch in Tank No. 1 which, by means of the connections on the control panel, causes the gates on Tank No. 2 to open. With the opening of the gates on Tank No. 2, this tank begins to fill, some of the storm sewage in Tank No. 1 flows back into the intercepting sewer and into Tank No. 2 until the level in the two tanks is the same, whereupon both tanks continue to fill. When both tanks are filled to the elevation of the overflow weirs, a contact is made in the pressure-switch in Tank No. 2, which, in the same manner, causes the gates on Tank No. 3 to open, and then this tank begins to fill. Some of the storm

sewage in Tanks Nos. 1 and 2 flows back into the intercepting sewer and into Tank No. 3 until the level is the same in all three tanks, after which the three tanks continue to fill together. When all three tanks have been filled to the level of the overflow weirs the excess storm sewage is discharged into the storm-overflow channel and thence to the river.

This method of operation was adopted in order to prevent all the tanks from being fouled by short storms of low intensity which do not produce sufficient excess storm sewage to fill all the tanks, and thus to reduce tank cleaning to a minimum.

Since the floors of the tanks are well above the invert of the intercepting sewer, as soon as the flow in the sewer above the tanks becomes less than the flow passing the regulator gates, the storm sewage remaining in the tanks slowly drains back into the sewer until the tanks are empty. The tanks are then cleaned, using the tank cleaners on the side walls to flush out deposited solids.

Emergency and Relief Over-Flows.—In the Control House, over the emergency outlet gates, are three emergency overflow weirs with crests at Elevation 35 so that, if all the gates on the stand-by tanks should be closed, and the emergency outlet gates should fail to open, sewage from the Olentangy-Scioto Intercepting Sewer above the regulator gates can back up and find a free outlet to the river. On the Alum Creek Intercepting Sewer, and on the connection from the Main Intercepting Sewer to the Control House, a relief overflow weir is located, with its crest at Elevation 16.75, so that, should the shut-off gates in the Control House be closed, sewage from the Alum Creek and Main Intercepting Sewers can back up and flow over this weir into the relief overflow conduit and thence through back-water gates to the river.

Alum Creek Storm Stand-By Tank

The Alum Creek Tank (Figs. 14 and 15) consists of a covered reinforced concrete tank, which, with a short length of 9-ft sewer, forms the connecting link between the Main Street Sewer and the Alum Creek Intercepting Sewer. The tank has a volumetric capacity below the crest of the overflow weir of 857 000 gal.

Along the west wall of the tank is a dry-weather flow channel which forms a continuation of the Main Street Sewer and through which, sewage flows during dry weather. The elevations of the invert of this channel and of the floor of the tank are such that, for all rates of flow to twice the estimated 1960 dry-weather rate of flow, sewage will be confined to the channel and will not flow into the tank. At the east end of the tank the excess storm sewage is discharged over an overflow weir into a storm-overflow channel and through a storm-overflow conduit to Alum Creek. A baffle in front of the weir prevents floating solids from escaping. On the copings of the north and south parapet walls of the walkways are tank cleaners of the type used at the Whittier Street tanks for flushing after a storm.

Due to the fact that the tank is covered, it was felt advisable to provide positive ventilation to be used whenever workmen enter for cleaning or in-

spection. This was done by installing two blowers in the Control House of sufficient capacity to give one change of air every 15 min when the tank is full to the crest of the overflow weir. Fresh air from the blowers is discharged into a duct in the west wall of the tank; it enters through openings

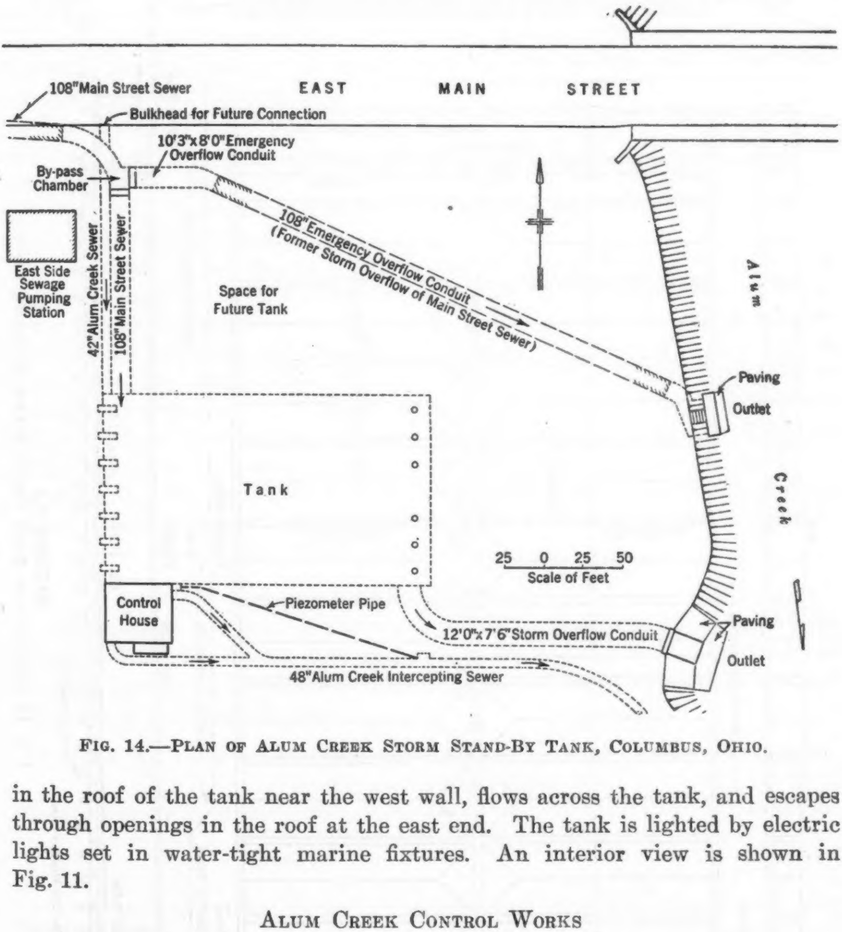


Fig. 14.—Plan of Alum Creek Storm Stand-By Tank, Columbus, Ohio.

in the roof of the tank near the west wall, flows across the tank, and escapes through openings in the roof at the east end. The tank is lighted by electric lights set in water-tight marine fixtures. An interior view is shown in Fig. 11.

Alum Creek Control Works

The control works consist of two electrically operated sluice-gates, a float-switch, and auxiliary electrical equipment.

Shut-Off Gate.—The shut-off gate, 48 in. square, permits shutting off the Alum Creek Intercepting Sewer so that no sewage from the Main Street Sewer can enter it. This gate is normally open.

Regulator Gate.—The regulator gate, 48 in. square, by means of a float control, automatically controls the volume of storm sewage from the Main Street Sewer passing through it so that the volume of sewage flowing in to the Intercepting Sewer is held to a predetermined maximum.

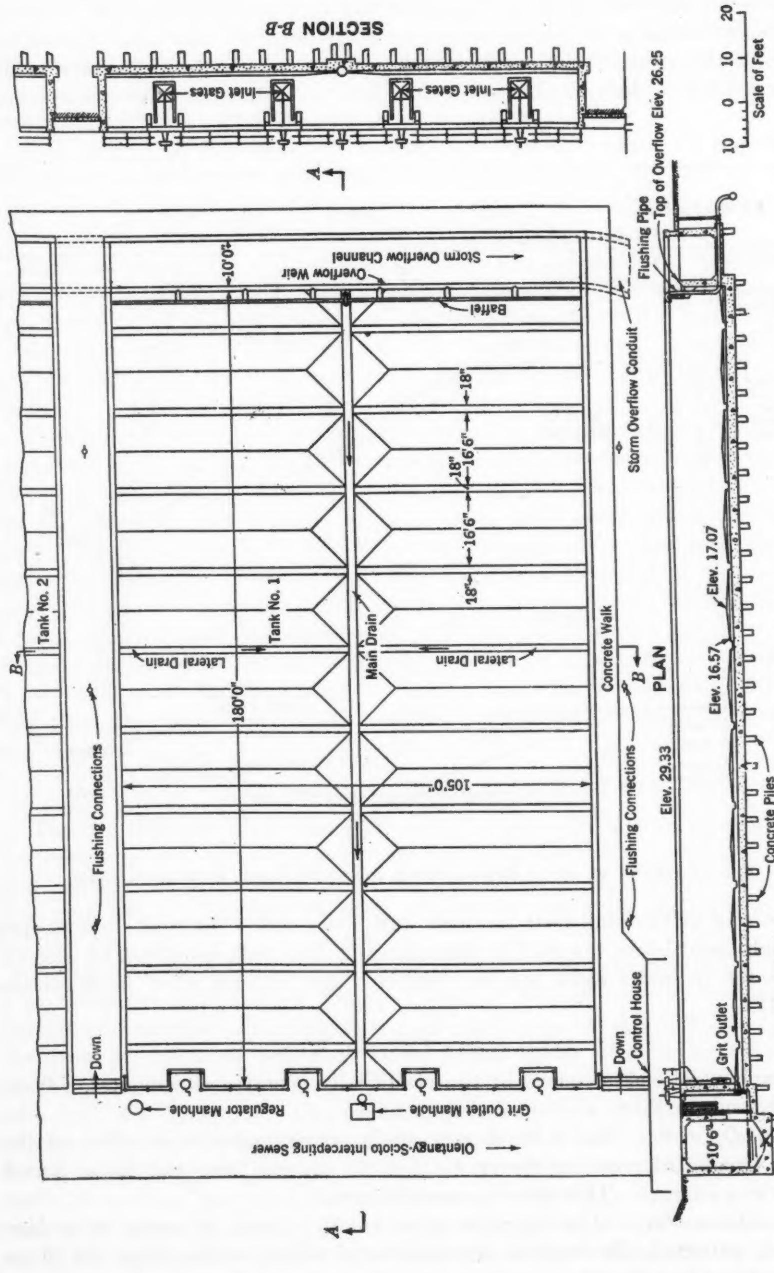


FIG. 15.—DETAILS OF ALUM CREEK STORM STAND-BY TANK.

Regulator Float-Switch.—The float-switch controlling the regulator gate is actuated by the rise and fall of the sewage in the intercepting sewer below the Control House. The electrical contacts are arranged so that the gate closes if the sewage level exceeds the predetermined elevation in the intercepting sewer below the gate; it opens if the sewage level falls below, but remains at rest while the sewage is at, this elevation.

Equipment in Control House.—The two blowers are electrically driven and have a total capacity of 13 400 cu ft of free air per min. The 3-ton crane is electrically operated.

Operation During Dry Weather.—When the Main Street Sewer is carrying only the dry-weather sewage flow, the shut-off gate and the regulator gate are open, and all the sewage flows in the dry-weather flow channel at the west end of the tank, through these gates, and thence into the Alum Creek Intercepting Sewer.

Operation During Storms.—During storms the regulator gate operates in the same manner as the regulator gates at the Whittier Street tanks. When the regulator gate comes into action, storm sewage in the dry-weather flow channel backs up and flows into the tank, and, as soon as the tank is filled, flows over the overflow weir and thence to Alum Creek. It should be pointed out that the former storm over-flow at the end of the Main Street Sewer has been converted to an emergency outlet for use in case of repairs or extensions to the storm stand-by tank, and that, in times of storm, all the flow from this sewer, other than that flowing through the regulator gate, passes into the Alum Creek tank. The method of cleaning is the same as that used at the Whittier Street tanks.

Cost of Work

Exclusive of the cost of engineering, the total construction cost of the works described was $3 621 000, as shown in Table 5, and, in Table 6, are given the costs per foot of, and the rate of progress on, the Olentangy-Scioto and Alum Creek Intercepting Sewers.

TABLE 5.—Cost of Intercepting Sewers, Regulator Chambers, and Storm Stand-By Tanks.

Structure (1)	Cost: Sub-total (2)	Cost: Total (3)
Intercepting Sewers:		
Olentangy-Scioto	$1 652 000	....
Alum Creek	939 000	$2 591 000
Regulator chambers	....	351 000
Storm Stand-By Tanks:		
Whittier Street	522 000	....
Alum Creek	157 000	679 000
Total cost	....	$3 621 000

TABLE 6.—Cost of and Rate of Progress on Olentangy-Scioto and Alum Creek Intercepting Sewers.

Type of sewer (1)	Size (2)	Length, in feet (3)	Average length of sewer completed per day, in feet (4)	Cost* per foot of completed sewer (5)	Remarks (6)
Olentangy-Scioto Intercepting Sewer: Rectangular					
A	10 ft. 6 in. by 5 ft. 6 in.	3 719	16	$45.09	Open cut
B	10 ft. 6 in. by 5 ft. 6 in.	590	16	57.13	Open cut
C	10 ft. 6 in. by 5 ft. 6 in.	520	5	207.93	Open cut
D	10 ft. 6 in. by 17 ft. 2 in. ±	681	4	86.02	Open cut
E	10 ft. 6 in. by 12 ft.	956	4	67.05	Open cut
F	10 ft. 6 in. by 12 ft.	250	4	87.24	Open cut
G	10 ft. 6 in. by 12 ft.	800	6	71.38	Open cut
H	10 ft. 6 in. by 12 ft.	2 334	6	80.35	Open cut
I	10 ft. 6 in. by 12 ft.	568	6	81.84	Open cut
J	10 ft. 6 in. by 12 ft.	239	5	95.88	Open cut
K	10 ft. 6 in. by 12 ft.	1 509	5	151.97	Open cut
Olentangy-Scioto Intercepting Sewer: Circular					
L	11 ft. 9 in.	2 092	6†	$93.26	Tunnel
M	10 ft. 3 in.	2 813	6†	71.29	Tunnel
N	9 ft.	2 242	12	32.25	Open cut
O	9 ft.	1 251	10	63.87	Open cut
Alum Creek Intercepting Sewer: Circular					
P	5 ft.	14 307	8†	$49.33	Tunnel
Q	5 ft.	611	4†	49.33	Tunnel
R	5 ft.	3 632	10	42.80	Open cut
S	4 ft.	3 096	26	14.41	Open cut

* Includes manholes, sub-drain, and other small miscellaneous items. † Per heading.

Engineering Organization

In the carrying through of the work described, the writers were assisted by John A. Rousculp, Jun. Am. Soc. C. E., and Robert T. Regester, Assoc. M. Am. Soc. C. E., Senior Designing Engineers, and by D. T. Mitchell, Harry Christiansen, J. T. Moore, and James H. Blodgett, Associate Members, Am. Soc. C. E., Field Engineers. They also wish to express their appreciation to the other members of the Engineering Organization for the real co-operation received from them at all times.

Acknowledgments

The works were begun under the administration of the Hon. James J. Thomas, Mayor, and the Hon. W. H. Duffy and the Hon. R. S. McPeak, Directors of Public Service, and were completed under the present administration, the Hon. Henry W. Worley, Mayor, and the Hon. W. P. Halenkamp, Director of Public Service.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PRE-QUALIFICATION OF CONTRACTORS

Discussion

BY C. J. TILDEN, M. AM. SOC. C. E.

C. J. TILDEN,[12] M. AM. SOC. C. E. (by letter).[12a]—In closing the discussion of this paper the writer wishes first to review briefly the way in which it came to be written. The Highway Division of the Society chose the subject as the topic for discussion at its meetings in 1930. Two meetings were held, one in April and one in October, at which papers were presented and discussed. Fifteen gentlemen stated their views either in writing or orally. The writer was asked to summarize this material and get it in shape for publication. A paper was published in *Civil Engineering*, for March, 1931, in which the real authors of the views expressed were named.[13] Once again the task was imposed of further condensing the article for publication in *Proceedings*. The present paper was the result. Even a foster parent has an affection for his child, and it is a source of great pleasure to the writer that so many have taken the trouble to discuss it.

Mr. Bush, who brings to the discussion the combined wisdom of an engineer and an official of a surety company, makes the interesting suggestion "to state in the call for bids that no bid will be received and opened from a bidder who has not qualified himself for the contract." This would mean a special pre-qualified list prepared in advance for each contract. He urges the desirability of having any scheme of qualification as simple and direct as possible so that good bidders will not be frightened off. He speaks further of the advantage of including in the list of basic elements determining the principles of pre-qualification some measure of the skill and ability of the contractor and the skill and experience of his organization, together with the information about other contracts which he may have in hand. These might be assumed to be included under the general heading of "previous experience"

NOTE.—The paper by C. J. Tilden, M. Am. Soc. C. E., was presented at the meeting of the Highway Division, New York, N. Y., January 22, 1931, and published in September, 1932, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: November, 1932, by Messrs. Edward W. Bush, A. B. Edwards, Samuel T. Carpenter, and Leon F. Peck; and January, 1933, by Messrs. C. L. Hall, A. R. Losh, and E. G. Walker.

[12] Strathcona Prof. of Eng. Mechanics, Yale Univ., New Haven, Conn.

[12a] Received by the Secretary September 25, 1933.

[13] "A Summary of the Present State of Prequalification," *Civil Engineering*, March, 1931, p. 504.

and "character and general reputation," but they are important enough to be mentioned specifically by themselves. Mr. Carpenter also stresses this point.

Mr. Peck quotes the stand taken by Maj. Cassius E. Gillette about twenty years ago, at a time when he was President of the American Society of Engineering Contractors, an organization which appears to be no longer in existence. The views which Major Gillette held in 1912, as a result of careful consideration of the whole question, are nevertheless interesting reading and worthy of note.

Colonel Hall has presented a singularly clear and able brief for the opposition; that is, emphatically against the principle of pre-qualification. In all, about twenty-two individuals have given their views, including those who presented the first group of papers and discussions. Of these, fourteen express themselves in favor of the general principle of pre-qualification; six are non-committal or deal in their discussion with other phases of the question; while only two are opposed. One of these is Mr. Peck, who quotes Major Gillette as being (in 1912) opposed to the principle; the other is Colonel Hall. Colonel Hall, then, is the emphatic representative of the case against pre-qualification. His discussion is so clear and excellent in every way that it should be read by all who are in any way concerned with the application of the principle. His conclusion, as stated in the last sentence of his discussion, is that "statutory pre-qualification would cause disadvantages outweighing its undeniable advantages."

On the other hand Mr. Losh thinks, that the advantages of pre-qualification are not brought out in sufficient detail in the original paper, and he proceeds to cite some of these advantages. He calls attention especially to the necessity of guarding against restricted competition.

Mr. Walker, writing from London, England, is mildly in favor of pre-qualification, but thinks it should be applied with modifications which he sets forth under three heads.

In general summary then, there seems to be a decided majority in favor of the basic principle of some form of pre-qualification of prospective bidders on public work. Just what form it should take appears to be subject to a rather wide range of opinion. Whether a generally satisfactory statement looking toward statutory principles could be written at this time is a matter of doubt.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

WORK OF RIVETS IN RIVETED JOINTS

Discussion

BY A. HRENNIKOFF, ESQ.

A. HRENNIKOFF,[54] ESQ. (by letter).[54a]—The response given this paper and the points of interest and importance brought out in the course of the discussion are gratifying. In answering these points it will be necessary to re-examine some of the original statements in the light of new information.

The main claim was that, in a riveted lap joint or butt joint with more than two rivets in a longitudinal row—under working conditions—distribution of the load among the rivets would be non-uniform, the outer rivets taking the greater part of the stress. Most of the discussers accept this assertion, and some of them mention experimental evidence in its support. However, correct theoretical estimation of this inequality and of the relative values of factors instrumental in producing it is almost impossible. The difficulty lies in the uncertainty of the relation between the slip of the plates at the rivet and the force developed by it. In a joint under a steady direct load this relation is determined mostly by the following three factors: The resistance of rivet in shear and bearing; the friction force; and the clearance around the hole caused by cooling of the rivet. Diversity of conditions and methods of fabrication are chiefly responsible for the uncertainty in relative importance of these factors in any particular joint; therefore, the force-slip curves assume a variety of shapes.

Messrs. Hill and Holt, referring to their experiments with double-strap butt joints of aluminum and steel plates, state that slip begins very early, that friction is comparatively unimportant, and that the force-slip curve does not depart from a straight line passing through the origin sufficiently to invalidate the results based on the straight-line relation.

The force-slip curve to which Mr. Larson refers (Fig. 15) is not straight. It begins vertically at the origin, indicating that very small loads are resisted by the rivet almost without any deformation, and then gradually diminishes its slope with the horizontal.

NOTE.—The paper by A. Hrennikoff, Esq., was published in November, 1932, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1933, by Messrs. Henry W. Troelsch Henry B. Seaman, A. H. Finlay, and F. P. Shearwood; and April, 1933, by Messrs. W. P. Roop, H. N. Hill and M. Holt, Donald E. Larson, and A. E. R. de Jonge.

[54] Instr., Dept. of Civ. Eng., Univ. of British Columbia, Vancouver, B. C., Canada.

[54a] Received by the Secretary September 5, 1933.

In the *Bulletin*[55] from which Fig. 15 was taken, several other force-slip curves of entirely different character are given, of which Fig. 17 is typical. This curve has a long, nearly vertical section, which changes its slope rather sharply into one not far from horizontal. Although the ordinates of this curve represent the average shear stresses for all four rivets in the joint, and not for the rivet under consideration, a moment of thought will convince one that if it was possible to reduce the ordinates to shear stresses of the rivet in question, the general shape of the curve would still be preserved.

Figs. 15 and 17 thus represent the types of force-slip curves that have come to the writer's notice, and he believes that most of the rivets in riveted joints, if tested, would exhibit one of these shapes or some intermediate form. (It may be mentioned, by the way, that Mr. Roop's Fig. 12, represent-

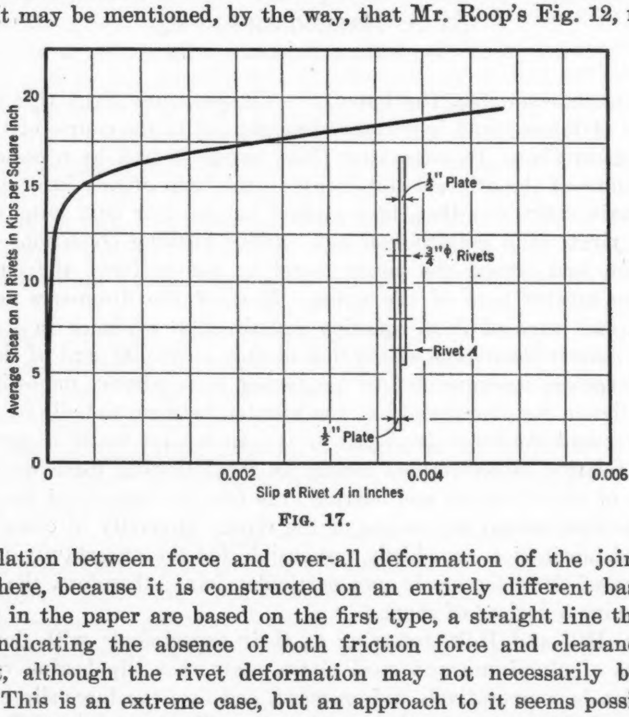


FIG. 17.

ing a relation between force and over-all deformation of the joint, cannot be used here, because it is constructed on an entirely different basis,) The formulas in the paper are based on the first type, a straight line through the origin, indicating the absence of both friction force and clearance around the rivet, although the rivet deformation may not necessarily be entirely elastic. This is an extreme case, but an approach to it seems possible under certain conditions, as is illustrated by the discussion of Messrs. Hill and Holt, who have found fair agreement between the experimental and the theoretical (based on an experimental value of the rivet coefficient) stress distributions among the rivets, as shown in Table 5.

When the force-slip curve deviates considerably from a straight line these formulas naturally do not apply; yet, if the expected force-slip curve is known, or at least the limits between which it may fluctuate, the distribution of the load among the rivets may nevertheless be obtained theoretically by the excellent method of successive approximations suggested by Mr. Larson.

[55] See Figs. 42, 45, and 48, in "Tests of Joints in Wide Plates," *Bulletin No. 239*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

The writer has applied the method to a few examples and has found that successive values of the rivet forces converge rapidly, so that the final results may be obtained after the second or third trial.

An analysis of a joint in which the rivets show the force-slip relation of the type represented in Fig. 15, still indicates an over-stress of the outer rivets. By using the curve of Fig. 17 the result may be different, and it seems appropriate to investigate this case in the light of the discussion presented by Mr. Shearwood, who strikes a rather different note, demonstrating how a friction force of a certain magnitude, combined with sufficient clearance around the rivet will cause equal stress distribution. The ideal stress-slip relation, according to Mr. Shearwood's assumption, seems to be as typified by Fig. 18. The rivet develops a force to the limit of its frictional

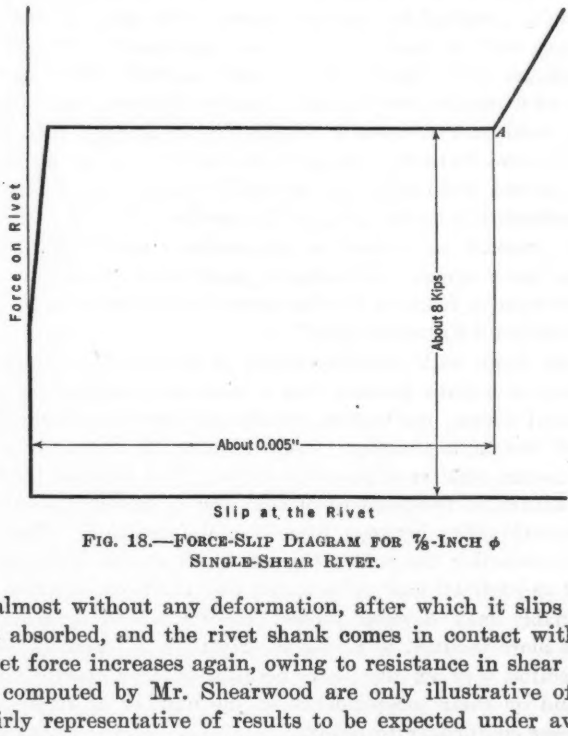


FIG. 18.—FORCE-SLIP DIAGRAM FOR ⅞-INCH ϕ SINGLE-SHEAR RIVET.

resistance, almost without any deformation, after which it slips until all the clearance is absorbed, and the rivet shank comes in contact with the plates; then the rivet force increases again, owing to resistance in shear and bearing. The values computed by Mr. Shearwood are only illustrative of course, but they are fairly representative of results to be expected under average conditions of fabrication. The resemblance which the curve of Fig. 18, from the origin to Point *A*, bears to the experimental curve, Fig. 17, is significant and testifies for the soundness of Mr. Shearwood's arguments. Since in this case a comparatively severe slip of the outside rivets requires no increase in rivet resistance, as long as any clearance is still left, Mr. Shearwood succeeds in demonstrating, by two examples of lap joint and butt joint, that if the working load is equal to the sum of the frictional resistances of all the rivets, the distribution of the load among the rivets will be perfectly uniform.

If it happens that the working load is less than the sum of the frictional resistances of all the rivets, a certain number of inner rivets are idle, the remainder exerting forces equal to their frictional resistances. On the other hand, if the working load is greater than the sum of the frictional resistances of the rivets the plates slip one over the other, and the excess of the load is borne almost completely by the extreme outer rivets.

Thus, summarizing the results for various kinds of force-slip relations, an approximate equality of rivet forces under working conditions seems possible only for one type of the force-slip curve (Fig. 17) and for one definite relation between the load and the frictional resistance, over which the designer has no control. For other conditions the working load does not divide equally, and the outer rivets are overstressed.

Referring to the diagrams constructed by Mr. Troelsch in which he compares the results obtained by his weld theory with those of the rivet theory, one would not fail to notice their close agreement. Of course, as Mr. Troelsch explains, this agreement is only natural, since the underlying assumptions of both theories—proportionality between the stress and deformation in weld and rivet—are identical. It is only fair to state that Mr. Troelsch's weld formula,[56] independent as it is of the number of rivets, if they are spaced uniformly, is preferable to the rivet formulas, which increase considerably in complexity as the number of rivets is increased.

Mr. Roop presents an interesting discussion elucidating the behavior of riveted joints under stress. Assuming a continuous connection between the plates, he arrives at a formula for the stress in the plates, σ, which is reducible to Mr. Troelsch's Equation (23).[56]

There is no doubt as to the correctness of Mr. Roop's statement that the clamping effect of a rivet extends over a large area around it; on the other hand, one cannot see any mechanism through the medium of which continuous stress over all the faying surface would assume the intensity at any point proportional to the relative slip at the point. The friction force caused by clamping is, naturally, independent of slip, and is indeed ruinous to a theory based on proportionality between force and deformation. That is why, in developing his formulas, the writer (by assumption) eliminated friction from consideration and left all load to be resisted by the isolated units (the rivets) which presumably may develop forces proportional to deformations. The writer is thus more inclined to prefer the rivet-stress formulas based on continuous connection between the plates because of their greater simplicity for long joints and on their independence of the number of rivets, rather than because of closer conformity to reality.

The assumption of rivets of very small size ("point rivets"), attributed by Mr. Roop to the writer, appears not absolutely necessary, although the finite size of the rivets naturally requires some reduction in the width of the plate, or in the pitch, similar to that suggested by the writer, in order to take account of the rivet hole; and also, as Mr. de Jonge points out, "the slip" at the rivet becomes actually the average slip, since different points in the

[56] *Proceedings*, Am. Soc. C. E., November 1932, p. 1503, Equation (15).

vicinity of the rivet slip somewhat unequally. Mr. Roop's discussion of the experimental phase of the question presents a valuable addition to his theoretical analysis.

To Mr. de Jonge is accorded the credit of contributing an extensive bibliography on the subject. No doubt it will be of great value to future investigators, especially to those who are acquainted with German. The writer wishes to thank Mr. de Jonge for calling attention to two assumptions used in the paper, but which were not stated: One concerning the meaning of the word, slip; and the other relating to the effect of disregarding the bending of lap joints, owing to their eccentricity. (The latter has also been noted by other discussers.)

Mr. de Jonge's way of attacking the problem of stress distribution among the rivets does not appeal to the writer. Introducing admittedly no new method, he uses an excessive number of unknowns; thus, $(6 \times 3 - 4) = 14$ equations would be required for the solution of a three-rivet lap joint, where the writer uses only two equations. This economy in the number of unknowns is largely due (aside from the assumption of symmetry) to the use of the displacement diagram, Fig. 3, which assists in forming a mental picture of respective rivets form the basis for the solution of other riveted joints. Answering a question raised by Mr. de Jonge, constructing such a diagram and equating the vertical distances between two of its curves to the slips of respective rivets, form the basis for the solution of other riveted joints.

The writer feels uncertain also as to the possibility of obtaining from Mr. de Jonge's equations a general solution, applicable to any number of rivets, unless such a solution would be so formidable as to be practically unworkable.

The justice of the main criticism advanced by Mr. de Jonge, that neither the subject-matter of the paper nor its method are entirely novel, must be partly conceded in view of the bibliographic data supplied by him; on the other hand, Mr. de Jonge joins the writer in his belief that the riveted joints "are not well understood," and this seems to be sufficient reason for the presentation of the present paper before the Engineering Profession. It is a significant fact in this connection that, according to Mr. de Jonge, only one theoretical investigation on the subject, that by Professor Batho, using a different method, had been published in English previous to this paper.

A brief discussion of Professor Batho's work[57] with the view of co-relating its results with those of the writer seems to be in order. In the theoretical part of his paper, Professor Batho uses the method of least work; and the algebraic expression for the work of deformation of a rivet in terms of the rivet force, X, is assumed to be proportional to KX^2, in which, K is a coefficient determined by experiment, and is constant for all rivets of the joint under a given load. The form of the rivet-work function, quadratic in X, involves a tacit assumption of proportionality between the rivet force and its deformation (which is equal to the slip of the plates). Under these circumstances, the resulting expressions for the rivet forces are exactly equivalent to the expressions derived by the writer.

[57] *Journal*, Franklin Inst., Vol. 182 (November, 1916).

In his experimental determination of the factor, K, for the double-strap butt joints, Professor Batho does not use the force-slip relation for rivets; his method of attack is to measure the strains in cover-plates between different rivets. This enables him to prove that the outer rivets are overstressed, and to find experimentally the actual distribution of the load among the rivets. In conjunction with his theoretical formulas this leads to an experimental value of the coefficient, K, as follows: $K = \frac{F}{100\,A}$, in which, F is the total force on the joint, in kips; and A is the cross-section area of the upset rivet, in square inches. This formula is intended to be applied only to the joints with dimensions used by the investigator.

The fact that F appears in the expression for K seems to contradict the assumed independence of K from the rivet forces, X, the sum of which is equal to F. Probably Professor Batho's result is due to the shape of the actual force-slip curves which in his tests apparently deviated from the tacitly assumed straight lines, in a manner similar to Fig. 15.

The theoretical relation between the writer's rivet factor, k, and the coefficient, K, of Professor Batho, can be proved to be $K = \frac{w\,t_2\,E}{p}\,k$, using the notation of the paper. Since k in Equations (26) and (27) is independent of the load, and K, according to Professor Batho, increases in proportion to the load on the joint, the two can agree only for one particular value of F. This value for some of the joints studied by Professor Batho is nearly equal to the working load; for others, it is considerably greater. Considering his expressions for k merely as first approximations the writer attaches no particular significance to these results, and mentions them only in view of questions raised by Professor Finlay and Mr. de Jonge. However, the fact that there is general agreement between the two investigations, conducted independently, and by different methods, is gratifying.

The writer wishes to correct an error in respect to the value of the rivet factor, k. As used in the paper, the coefficient, k, in the double-strap butt joint represents the ratio of the rivet deformation to the force transmitted to one splice-plate, and not to both splice-plates, as apparently assumed by Messrs. Hill and Holt. This means that their values of the coefficients in Table 4, if used with the formulas of the paper, must be multiplied by 2. Then, the theoretical value comparable to the experimental value of 0.00012 in. per kip mentioned by Messrs. Hill and Holt will be 0.000091 in. per kip (one-half the value found from Equation (26)), which is somewhat smaller than the experimental.

The writer quite agrees with Messrs Shearwood, Roop, and de Jonge when they state that the behavior of a riveted joint under variable load depends on its entire previous stress history; but he does not attempt to discuss here this phase of the question, limiting himself to a consideration of the action of a steady load and to the task of calling attention to the weaknesses of conventional design, the assumptions of which (using an apt expression by Professor Finlay), "if given sufficient time for their assimilation, tend to be confused with the eternal verities."

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DISCUSSIONS

DISTRIBUTION OF SHEAR IN WELDED CONNECTIONS

Discussion

BY HENRY W. TROELSCH, M. AM. SOC. C. E.

HENRY W. TROELSCH,[19] M. AM. SOC. C. E. (by letter)[19a].—In preparing the analysis presented in this paper, the writer purposely made the underlying assumptions as simple as possible. Any other procedure would have involved complicated mathematical expressions, which would have tended to obscure the results. However, for stresses within the elastic limit, the theory offers a picture of the distribution of shear in longitudinal fillet welds which is qualitatively correct. Whether it can be made to yield numerically correct values of the shear in various parts of the welds can be determined only by checking against the results of tests. Such tests as have come to the writer's attention indicate that the distribution of shear can be closely represented by Equation (15), provided the proper values of the detrusion ratio, D, and the areas, a_1 and a_2, are used.

The closeness of the correspondence between test and theory is shown in Fig. 16, plotted from the results of two tests selected from those made by the late J. Hammond Smith, M. Am. Soc. C. E., at the University of Pittsburgh, Pittsburgh, Pa.[20] The plotted points show the measured slip between gauge points on the bars. In Fig. 16(b) the curve is plotted from the equation,

$$q = \frac{v}{D} = \frac{P b}{a_2 \, E} \; \frac{\cosh \dfrac{x}{b}}{\sinh \dfrac{x_1}{b}}$$

using for a_1 and a_2 the actual area of the bars, and for D the value, 7 100 000, which is required to make the curve fit the plotted points.

NOTE.—The paper by Henry W. Troelsch, M. Am. Soc. C. E., was published in November, 1932, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1933, by Messrs. F. T. Llewellyn, A. S. Woodle, Jr., Milton Male, William Hovgaard, Charles W. Chassaing, and F. E. Fahy; March, 1933, by W. H. Jameson, Assoc. M. Am. Soc. C. E., and April, 1933, by Messrs. Isamu Ohno, and P. Wilhelm Werner, Assoc. M. Am. Soc. C. E.

[19] Designing Engr., Am. Bridge Co., New York, N. Y.

[19a] Received by the Secretary September 21, 1933.

[20] "Stress-Strain Characteristics of Welded Joints," by J. Hammond Smith, *Journal*, Am. Welding Soc., September, 1929, p. 79.

In Fig. 16 (*a*), the plates are wider, and the effect of the non-uniform distribution of their tension, to which Mr. Llewellyn and Mr. Male have referred, becomes evident. To make the formula agree with the test results it is necessary to reduce the values of a_1 and a_2 from the actual area, 3.0, to 1.59 for a_1 and 2.44 for a_2. The detrusion ratio, D, is 5 480 000. The amount of reduction in the areas of the plates is less than that obtained by Messrs.

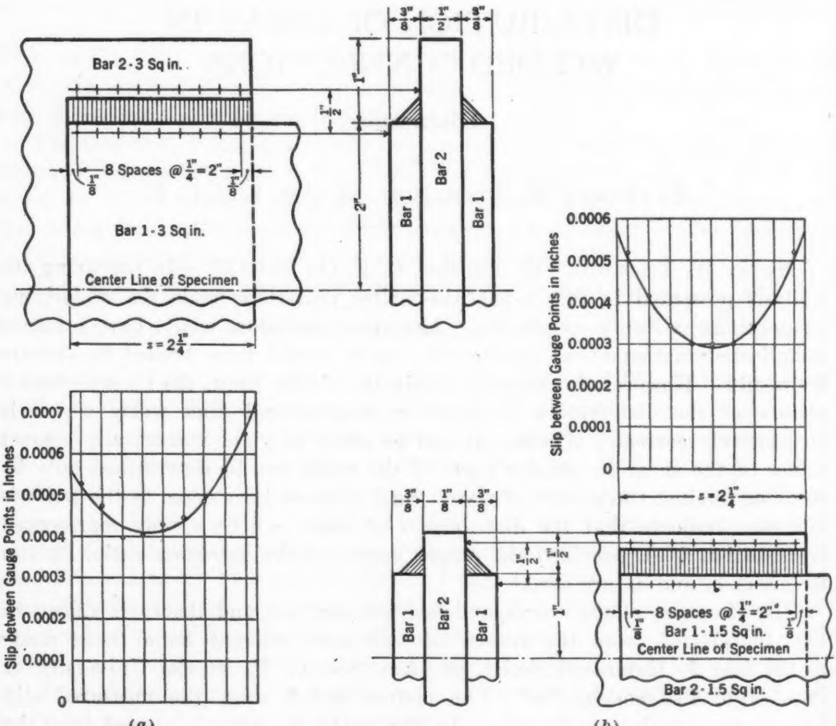

FIG. 16.

Weiskopf and Male.[3] It is to be noted, however, that the results agree with the findings of Messrs. Weiskopf and Male to the extent that the reduction for Bar 1 is greater than that for Bar 2.

The detrusion ratio obtained from these tests is not that for the welds alone, but for the welds plus adjacent parts of the bars. For purposes of analysis it is impossible to say where the weld stops and the plate begins. It may be that an analysis, which, like the foregoing, is based on "equivalent areas" and a slip between points near the welds, is as near a practical solution as can be expected.

The fact that the theory, with such modifications as have been suggested, shows close agreement with measured deformations, indicates that it is sub-

[3] "Stress Distribution in Side-Welded Joints," by W. H. Weiskopf, Assoc. M. Am. Soc. C. E., and Milton Male, Jun. Am. Soc. C. E., *Journal*, Am. Welding Soc., September and December, 1930.

stantially correct. The theories suggested by Mr. Woodle and Professor Chassaing, based on a different set of fundamental assumptions, show no such agreement with the tests.

It is to be observed that in both these tests the unit shears for which the deformations were measured, are such as would be applied to similar welds in designing practice. The average shear in the welds is 2 670 lb per lin in. of $\frac{5}{16}$-in. fillet weld.

It is undoubtedly true, as Mr. Llewellyn and Mr. Male have pointed out, that welded joints will carry, without failure, loads which according to this theory would stress the ends of the welds beyond the breaking point. This can be explained by assuming that after a weld has been stressed to the yield point, further deformation occurs without much increase in stress, so that for a considerable distance near each end of the joint the welds are stressed to the yield point or slightly beyond it. A joint in such a state of stress will probably carry its load satisfactorily if the load is constant. On the other hand, as has been pointed out[21] by H. M. Priest, M. Am. Soc. C. E., impact or variable loads, and especially loads which alternate between tension and compression, might lead to a fatigue failure because of repeated over-stressing of the ends of the welds. Inasmuch as variable and alternating stresses occur regularly in bridge trusses, an experimental investigation of the behavior of welded joints under the action of such stresses should yield valuable information.

Mr. Werner's analysis of a joint with tapered bars indicates that to obtain anything approaching a uniform distribution of shear, the bars must be tapered nearly to a knife-edge; that is, k must be nearly zero; for in Mr. Werner's Example 2, with k as small as one-fifth, the end shear is still 2.5 times the average shear. The fact that a taper tends to eliminate excessive local stresses at an abrupt change in section has long been recognized in the practice of finishing re-entrant angles of machined parts with a fillet.

Mr. Ohno's application of the theory to a reinforcing plate is along the line suggested by Professor Hovgaard in connection with longitudinal girders or deck-houses welded to the deck of a ship. It should be possible to make a similar application to the cover-plates of a plate girder, although the mathematical analysis will be complicated by the variation of the flange stress along the length of the girder.

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[21] "The Practical Design of Welded Steel Structures," by H. M. Priest, *Journal*, Am. Welding Soc., August, 1933.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

MODEL LAW FOR MOTION OF SALT WATER THROUGH FRESH

Discussion

BY MORROUGH P. O'BRIEN AND JOHN CHERNO, ASSOCIATE MEMBERS, AM. SOC. C. E.

MORROUGH P. O'BRIEN[14] AND JOHN CHERNO[15], ASSOC. MEMBERS, AM. SOC. C. E. (by letter)[15a].—In regard to Lieut. Vogel's suggestion that the symbols used in model work be standardized, the writers agree heartily. The symbols in this paper were changed to conform to those of the Society's Special Committee on Irrigation Hydraulics, in so far as was possible.

The experiments under discussion were made for two purposes, as follows: (1) To study the motion of salt water through fresh water, particularly with reference to the dilution of the salt water during the process; and (2) to discover the model law for this phenomenon which was then to be used to design a model of a part of San Francisco Bay.

This paper is concerned only with the second objective, and whether the water surfaces were equalized on the two sides of the gate does not affect the validity of the resulting model law. Mr. Grunsky is not correct in assuming that lock-gates or lock operation was being studied in these tests. Conditions in actual locks are further complicated by the overfilling of the chamber resulting from the inertia of the water in the culverts; by the non-uniform distribution of salinity in the lock chamber; and by the residual motion of the water in the lock resulting from the filling process. While none of these factors is of major importance in the general characteristics of the motion after the gate is opened, they may disturb the motion in an actual lockage, and the formulas given in the paper must not be used blindly.

The writers cannot agree to Mr. Grunsky's "substantial equilibrium" in the brackish waters of an estuary (disregarding tidal effects) unless the word, "substantial," is changed to "dynamic" or "statistical." The surface

NOTE.—This paper by Morrough P. O'Brien and John Cherno, Assoc. Members, Am. Soc. C. E., was published in December, 1932, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1933, by W. E. Howland, Assoc. M. Am. Soc. C. E.; April, 1933, by Messrs. Herbert D. Vogel, and C. E. Grunsky; May, 1933, by John B. Drisko, Jun. Am. Soc. C. E.; and August, 1933, by G. H. Keulegan, Esq.

[14] Assoc. Prof., Mech. Eng., Univ. of California, Berkeley, Calif.

[15] Sacramento, Calif.

[15a] Received by the Secretary August 29, 1933.

gradient would cause water near the surface to flow down stream and the excess pressure due to the salt water would cause an under-flow up stream. The salt water, being diluted with the overlying fresh water, will become less and less brackish during its up-stream movement and, at the same time, the fresh water moving down stream will increase in salinity. As much of the surface gradient as is not used to move the net river flow will serve to maintain these two opposed currents.

In a tidal estuary it is difficult to measure these currents, but it is known from current meter measurements that "the flood tide comes in at the bottom;" that is, the ebb current turns to flood current sooner at the bottom than at the surface, the time lag being as much as two hours in some cases. This agrees with the conception of two separate and opposed currents. The surface gradient in a tidal estuary resulting from varying salinity cannot be obtained by simply comparing the elevation of mean sea level at two points because a gradient would exist also with no river inflow, and with constant salinity. This gradient must be great enough to discharge the tidal prism at ebb tide, and this same surface slope tends to decrease the volume of the flood tide. The measured gradient in a tidal river comprises the gradients resulting from net discharge, varying salinity, and tidal flow. Mean sea level, as defined by the U. S. Coast and Geodetic Survey, at Stockton, Calif. (92

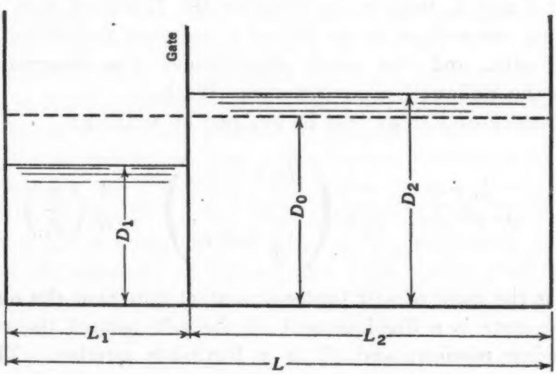

Fig 18

miles from the Golden Gate) was 1.5 ft above mean sea level at the Golden Gate in August, 1930; in the same month, mean sea level at Dumbarton Bridge (33 miles from the Golden Gate), was 0.4 ft above the same datum in spite of the fact that there is practically no variation in salinity and no fresh-water inflow in the southern arm of San Francisco Bay.

Some experiments were made that have a bearing on Mr. Drisko's question regarding the effect of different water-surface elevations on the two sides of the gate. Referring to Fig. 18, the surfaces will ultimately come to rest at a depth, D_0, given by,

$$D_0 = \frac{1}{L} (D_1 L_1 + D_2 L_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

If the gate is moved to the left (for the conditions shown) until this depth, D_0, is obtained, the length of the lock will be,

$$L_0 = L_1 \frac{D_1}{D_0} \quad \text{.......................} (34)$$

If the modulus, K, is computed from these values of L_0 and D_0 and the actual S_0, the velocity curve drawn using Fig. 13 agrees quite well with the experimental values, particularly during the later stages of the motion.

In reply to Mr. Howland's inquiry, the surface slope shown in Fig. 6 existed at the heads of the fresh-water and salt-water waves. Between the two, the interface was practically horizontal provided the lock was long enough (compared with its depth) to make this surface form possible for any length of time. After fresh water reached the closed end of the lock, the salt-water slug had the form shown in Fig. 7. The center of gravity of the salt water moves toward the right, but the edges of the salt slug tend to spread in all directions from the center of gravity. Spreading at the front edge is resisted by the internal friction, but spreading backward reduces the velocity, thus giving the peculiar shape shown. After this shape is reached, the motion does not continue primarily under the action of a pressure gradient, as suggested by Mr. Howland, but rather on account of inertia, until kinetic energy is dissipated.

Motions at C and E, such as described by Mr. Howland, were not observed. Floats placed on the surface to the left of C acquired full velocity toward the left very abruptly, and the same phenomenon was observed by one of the writers in the Ballard Locks, at Seattle, Wash.

The significance of K may best be grasped by writing:

$$K' = \frac{L_0\, \nu}{g^{0.5}\, s^{0.5}\, d^{2.5}} \propto \frac{\nu}{V_0\, d} \left(\frac{L_0\, d\, \frac{w}{g}\, s}{\frac{1}{2}\, w\, d^2\, s} \right) = \frac{1}{R} \left(\frac{M_s}{F_s} \right) \text{.........} (35)$$

in which, M_s is the mass of salt (not salt water; note that the only net effect of opening the gate is a displacement of the salt toward the right); F_s is the force causing motion; and, R is a Reynolds number. The Reynolds number indicates the relative importance of inertia and friction forces, large values of R corresponding to motions predominated by inertia effects and the reverse. The factor, K', therefore, becomes large when a large mass of salt is to be moved by a small force under conditions conducive to large frictional resistance. The curves corresponding to large values of K in Fig. 7 show a rapid decrease in velocity for precisely this reason. (In tests of this type larger dilution values can be obtained if K is small.)

It should be pointed out here that μ was not defined as the absolute viscosity (see following Equation (10)), but as a coefficient of internal friction and that the conclusions state that the internal friction behaves in a manner similar to the viscous force of laminar flow, but nothing was said as to the value of μ or ν. The theory of turbulent flow as developed by Prandtl, von Karman, and G. I. Taylor expresses the frictional resistance in terms of

transfers of momentum or vorticity which results in a "mechanical viscosity" having properties similar to the viscosity of laminar flow. In fact, in the original draft of the paper, ν was called the "mechanical viscosity," but the term was dropped to avoid lengthy explanation of its significance. The important point is that the dilution or mixing is an integral part of the mechanism of turbulent friction and the existence of turbulent mixing was assumed throughout. A model law which represents the motion accurately, particularly the dissipative effect of internal friction, must also represent the mixing. For this reason, the use of mercury in a model is perhaps inadmissible, but the point should be investigated.

Dr. Keulegan's more rigorous derivation of the model law from the standpoint of energy is interesting and it is unfortunate that the accuracy of the experiments does not warrant a comparison of this model law with the simplified law proposed by the writers. Equation (5) was originally derived from the standpoint of energy, and this derivation is perhaps preferable to the momentum method, which is open to criticism from a mechanical standpoint.

Mr. Howland suggests that the motion be considered as the result of the action of a force equal to $\frac{1}{2}\, w\, s\, d^2$ tending to move the entire mass of water between Sections *C-C* and *E-E* toward the right, and infers that this tendency must be accompanied by a vertical movement of the free surface. Apparently, he overlooks the fact that the salt water to the left of *C* has a downward acceleration which reduces the pressure below the hydrostatic pressure corresponding to the depth. Pressure adjustments are propagated with the velocity of sound, which in the present problem is equivalent to occurring instantaneously. As soon as motion of salt water begins, vertical accelerations at *C* reduce the pressure and the fresh water begins to move toward the left.

In obtaining Equation (2), the force acting was based upon the difference in density and the momentum was taken as the arithmetic rather than the vector sum of the momenta of the salt and fresh water, even though the two are oppositely directed. This is necessary because the liquid between Sections *C-C* and *E-E* is not a free body in the ordinary sense, but rather the salt or fresh water moves subject to the restraint imposed by the presence of the other fluid. The condition is entirely analogous to the oscillations of liquid in a **U**-tube manometer. The activating force results from the unbalance of the columns but the momentum is the arithmetic sum of the momenta in the two legs and not the vector sum. If the vector sum of the momenta was taken in Equation (2), the initial velocity, V_0, would be found to depend only on the depth and the acceleration of gravity and would be independent of the difference in density. Equation (5) can also be obtained by considering the salt (not salt water) as a free body of weight, w, s, and mass, $\frac{w}{g}\,(G_1 + G_2)$ per cu ft. This method is reasonable since the net effect of opening the gate is simply a transfer of salt toward the right.

The writers wish to thank the various discussers for their valuable contributions and to offer to them, and to others who are interested, any original data which were not included in the paper.

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DISCUSSIONS

METEOROLOGICAL DATA
PROGRESS REPORT OF SPECIAL COMMITTEE

Discussion

BY MESSRS. FLOYD A. NAGLER, AND JOHN W. PRITCHETT

FLOYD A. NAGLER[37], M. AM. SOC. C. E. (by letter)[37a].—The writer has never undertaken a study of the hydrology of a project for which the available meteorological data were all that could be desired. Invariably, there has been a deficiency of some kind, either in the length and continuity of the records, too wide a scattering of stations, or a lack of specific information dealing with rainfall duration and intensity. No one should criticize the U. S. Weather Bureau organization for shortcomings of this nature, since they are generally due to the lack of financial support necessitating a continual struggle to overcome the restrictions placed upon its service by exceedingly low budgets. At this moment (1933) Weather Stations that have given continual service for half a century are being discontinued under a plea of Government economy.

The U. S. Weather Bureau should also have sufficient funds and personnel to make studies and analyses of data in its possession. The Miami Conservancy District made an invaluable contribution in its published analysis of rainfall data in Eastern United States, but why should individual projects be burdened with the cost of general studies of this nature? The writer has knowledge of several very useful studies made by meteorologists of the U. S. Weather Bureau, mostly on their own initiative and during spare time, but the Government has never been able to publish these manuscripts due to an inadequate printing budget.

In spite of some demand to the contrary, the practice of the Bureau in arranging meteorological data according to the calendar year should be endorsed. In grouping stream-flow data according to an arbitrary water-year, the U. S. Geological Survey has initiated a procedure that has resulted in

NOTE.—This Progress Report of the Special Committee on Meteorological Data was presented at the Annual Meeting, New York, N. Y., January 18, 1933, and published in January, 1933, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: April, 1933, by Messrs. C. S. Jarvis, C. F. Marvin, and Ivan E. Houk; May, 1933, by C. E. Grunsky, Past-President Am. Soc. C. E.; August, 1933, by Messrs. Charles W. Sherman, J. C. Stevens, Thorndike Saville, A. L. Sonderegger, and Edward Hyatt; and September, 1933, by Messrs. Joseph Jacobs, and J. A. Fraps.

[37] Prof. of Hydr. Eng., Univ. of Iowa, Iowa City, Iowa.

[37a] Received by the Secretary September 18, 1933.

more confusion than merit. Not only does the climatic or water-year differ in various parts of the country, but there is great variation from year to year, some years having a length of fourteen months, while others may be only nine or ten months in duration. In a large part of the country, November 1 rather than October 1 is a better date for the average beginning of the water-year. It is to be hoped that meteorological data will continue to be arranged according to the calendar year and that stream-flow data will again be published in this manner, allowing those few who desire to make studies upon a different basis to re-arrange the data to suit the issue and locality.

JOHN W. PRITCHETT,[38] ASSOC. M. AM. SOC. C. E. (by letter).[38a]—Considering the recommendations of the Committee, made in order to develop greater service on the part of the United States Weather Bureau to the Engineering Profession, Recommendation 2 is of special importance.[39] The bulletin recommended should give a complete history of each station (as far as may be), showing errors that have been corrected in previously published data, changes in locations of stations and dates of changes, and any other historical information that may be of assistance to those making use of such data.

As an illustration, "Climatological Data, Texas Section, January, 1933," shows for the Crosbyton Station a record of forty-seven years. From 1886 to 1891, the station was at Silver Falls, near Mount Blanco, Tex., about twelve miles north of the present Town of Crosbyton.[40] From 1892 to June, 1916, the records were taken at Mount Blanco. In January, 1917, the station was moved to Crosbyton, where the records are now (1933) being taken. All these records were combined with those made at Crosbyton. In the 1920 and 1930 monthly "Summary of Climatological Data by Sections," a note is given, stating that from April, 1886, to December, 1891, the values given are for Silver Falls. However, any one taking data from the present monthly report or from the annual summary would naturally conclude that the record represented a 47-year record at Crosbyton.

In connection with the history of this record, a letter from the Meteorologist in charge of the Section Center at Houston, Tex., states that the observer at Mount Blanco, for the first eight years of his service, made his daily entries of precipitation without decimal points, and apparently recorded the amounts to the nearest hundredth of an inch; but in August, 1905, he called the attention of the Section Center—then at Galveston, Tex.—to the fact that his measurements were made to the nearest tenth of an inch, only. The Station records were then corrected to give the true amounts, but the printed records held as surplus were not corrected; consequently, there were numerous errors in such records for Mount Blanco prior to 1906. It is important that such information be made available to those using climatological data.

[38] Office Engr., State Board of Water Engrs., Austin, Tex.

[38a] Received by the Secretary September 18, 1933.

[39] *Proceedings*, Am. Soc. C. E., January, 1933, p. 155.

[40] Sen. Ex. Docs., 52d Cong., 1st Session, 1891–92, Vol. 2, "Climatic Conditions of Texas."

The collection and publication of long meteorological records now kept privately, also, of such records already made, or being made, by various other Federal and State departments, together with the history and descriptions of the stations, would be very valuable.

The publication of climatological data by units or districts as now divided is convenient. However, these district boundaries have not always been the same. As a result of changes in district or section boundaries, it is sometimes difficult to trace the record of a particular station. The publication of all past records by districts as now established would be of material assistance in compiling climatological data.

Regular inspection of co-operative stations by inspectors for the Weather Bureau would be a valuable service. Errors such as those noted herein would soon be detected and corrections made. Valuable instructions would be given to the observers, and interest in the work would be stimulated.

The initiation, by the Bureau, of a program of research which will allow the development of a correcting factor or relationship that can be applied to records obtained from instruments exposed above ground level, or otherwise exposed so as not to reflect actual conditions, is of much importance. The investigations should go far enough to determine such factors or relationship, or to determine, satisfactorily, that such investigation is not practical.

A program of co-operative research with many scientific institutions throughout the United States is desirable. The Committee states[41] that "stations reporting river and flood information are grouped into districts chiefly by rivers and their immediately contiguous water-sheds, each with a central station within the district, although on large rivers the work is divided among several districts." It is suggested that a grouping into districts conforming to the United States Geological Survey Water Supply Districts might be more convenient for many purposes.

The Weather Bureau is to be commended for the long successful service and the vast amount of useful data assembled by the co-operative observers. The present demand for further publication, and a more practical arrangement, of these data serves to indicate the more extended use now being made of them. The distribution of these stations necessarily depended upon the settlement and growth of the rural sections. A further increase in the number of stations in some sections of Texas would be very desirable. Such an increase is more to be desired than extreme accuracy of observation and records. The accuracy of most rainfall and temperature records is well within the range of variation of rainfall and temperature from place to place. It is well known that in the Southwestern Section of the United States several severe showers may occur over a particular area, amounting to as much as $\frac{1}{4}$ in., or even as much as $\frac{1}{2}$ in. in depth, while at other points not more than a mile distant no rain falls. Such differences in rainfall are more likely to occur during the spring and summer seasons. Almost any rainfall record for a given locality is better than an estimated rainfall based on records of a distant station.

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[41] *Proceedings*, Am. Soc. C. E., January, 1933, p. 164.

Many long and reasonably accurate rainfall records are being kept privately that could be made available to the public through the Weather Bureau, if the Bureau was allowed to appoint inspectors to visit co-operative stations and to collect such data. Many records that have never been published, have also been kept by other Federal and State agencies.

The desirability of an increase in the number of stations applies to the co-operative stations for the recording of rainfall and temperature. Possibly there are enough regular Weather Bureau Stations to conduct the necessary investigations, carry on experimental work, and record the other climatological data.

Some engineers feel that the Weather Bureau should publish its data for the climatic year rather than for the calendar year as at present. No climatic division could be found which would be applicable to the entire United States. It is usually necessary to copy, re-arrange, and combine climatological data before they can be made use of in engineering investigations. The data are more easily found and assembled in their present form than would be the case if some arbitrary division were made.

Attention has been called to the infrequency of calculations of new normals for temperature, rainfall, etc. Available information should be at hand, showing how such normals were derived, and the data of calculation.

The Weather Bureau has rendered and is rendering a great service. Valuable records are being made which may be of increased service to the public through a carefully planned program of improvement. The past success of the Bureau is due, in a large measure, to the long and faithful service of its members, from the co-operative observers to its present Chief. The long, faithful, and efficient service rendered by him is worthy of commendation.

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DISCUSSIONS

DEVELOPMENTS IN REINFORCED BRICK MASONRY

Discussion

By Messrs. J. W. McBurney and E. G. Walker

J. W. McBurney,[28] Esq. (by letter).[28a]—The author has performed a service in making generally available, and in correlating, certain hitherto unpublished data on reinforced brick masonry tests.

Mr. Hansen gives averages for the ratio of compressive strength of brick flatwise to compressive strength on edge, modulus of rupture to compressive strength flatwise, etc. He states, "however, there is a considerable variation of these ratios for brick in different territories" (see heading, "Compressive Strength"). It may be of interest to know that the ratio of compressive strength flatwise to compressive strength on edge ranges from 0.74 to 2.30 and the ratio of modulus of rupture to compressive strength flatwise has a range of values from 0.070 to 0.426.[29] It should be emphasized that most of this variation in ratio is not due to chance or error. A particular type of brick is usually characterized by a rather constant and definite ratio. As the writer has stated previously,[30] the fact that a given ratio may be the average for a very large number of tests does not increase the probability that the average ratio will represent the actual ratio for a particular kind of brick.

This same point of view should apply to reporting compressive tests on brick masonry. Referring to a paper of which the writer was a co-author, Mr. Hansen credits the Bureau of Standards with reporting that the average strength of wallettes, 18 by 34 in. high with 1:0.1:3 cement mortar, was 2 110 lb per sq in. To secure this average, he includes items given in the report that belong in separate categories, and, therefore, his statement is open to question.

The writer disagrees also with the statement that "it is generally conceded that walls 9 ft high will develop a compressive strength equal to from 25 to 30% of the brick strength." Considering brick strength, mortar, workmanship, and regularity of size and shape of the brick units as factors,

Note.—The paper by James H. Hansen, Jun. Am. Soc. C. E., was published in March, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1933, by Messrs. W. K. Hatt, and C. T. Schwarze.

[28] With Bureau of Standards, U. S. Dept. of Commerce, Washington, D. C.

[28a] Received by the Secretary May 1, 1933.

[29] "The Compressive and Transverse Strength of Brick," by J. W. McBurney, *Research Paper No. 59*, Bureau of Standards, p. 834.

[30] *Loc. cit.*, p. 833.

the authors of *Research Paper No. 108* reported ultimate compressive strength of brick masonry expressed as percentages of brick strength, ranging from 8.8 to 44.0%, restricting the comparison to clay bricks laid as solid 8 and 12-in. walls.

It is possible that the man who runs as he reads may not understand the implications of "best brick masonry" referred to in the statement of the recommendation of the Department of Commerce Building Code Committee. The working stress of 600 lb per sq in. is limited to the following combination of factors: (1) The brick must be at least 8 000 lb per sq in. in compressive strength tested flatwise; (2) the mortar must be Portland cement mortar as defined by the Code; (3) the workmanship must be thoroughly inspected and must be such as to produce smooth, level, horizontal joints and completely filled vertical joints; and (4) the effects of eccentric and concentrated loads and lateral forces must be fully analyzed and allowance made for them. For certain other combinations of factors, the Committee limits the working stress to 50 lb per sq in.

It is suggested that mortars with a cement-lime-sand ratio of 1:0.15:3 be referred to as cement mortar with an addition of lime. There is considerable evidence that the strength of a 1:3 cement mortar is not impaired by the addition of not more than 0.25 parts of lime by volume. Therefore, the practice is recommended of limiting the term, cement-lime-mortar, to compositions ranging between 1:0.25:3 and 1:2:9, of which 1:1:6 is typical.

In conclusion, the hope is expressed that reinforced brick masonry has emerged from its demonstration phase.

E. G. Walker,[51] M. Am. Soc. C. E. (by letter).[51a]—The author has condensed into the compass of a short paper an excellent summary of the recent American experimental work on reinforced brickwork. In tracing the historical development from the time of the first Brunel, he stops short at the 1851 Exhibition, and leaves the impression that nothing has been done in Great Britain since that time. In fact, the practice of reinforcing brickwork in an empirical manner by the use of flat iron strips or round iron rods has been in use ever since the early days described by the author. The use of the hoop-iron bonding, which consisted of strips of flat iron hoops laid continuously in the horizontal mortar beds at vertical intervals of a few courses, has been, and is, a well-known method of binding together brick structures. Reinforced brickwork with round rods has also been developed, and structures similar in character to that illustrated in Fig. 4 are not uncommon.

Contracting firms have specialized in this class of work. Although it is not correct to state that the art of reinforcing brickwork was forgotten, nevertheless, the broad conclusion is true that the development of reinforced concrete construction, which first began in Great Britain on systematic lines about 1898, has retarded a development in brick construction which probably would have taken place otherwise. The reinforced brickwork that has been built during the Twentieth Century has been more usually in the form of

[51] (Maxted & Knott), London, England.

[51a] Received by the Secretary June 10, 1933.

vertical walls, panels, and columns which, by being reinforced, have been made thinner. Of the more scientific use of the material for beam, slab, arch, and similar constructions of kinds common in reinforced concrete, there have not been many examples, even though the pioneering work was done in and around London by men famous in the annals of British Civil Engineering.

In India, conditions are quite different. Throughout Southern Asia the brick has been the pre-eminent basis of permanent construction from the earliest recorded times. British engineers in India, who must utilize local material and local labor for their work, have thus turned naturally to the possibilities of brickwork rather than concrete. Until about 1913, all cement used had to be imported into India. The well system of constructing foundations, so preponderatingly used in India, is an excellent example of the supremacy attained by brickwork there.

In countries such as the United States and Great Britain, brickwork does not start with any such advantages as it does in India. Indeed, the opposite is the case. Concrete-making materials are easily attainable in most parts of these countries; uniform high quality is general; and the machinery and technique of concrete making and placing are highly developed. Pouring concrete, on a job of any size except the smallest, is mainly a mechanical process involving little direct labor beyond the spreading, spading, and ramming of the concrete in the forms. Considerable hand labor must be employed, however, on many jobs, in making and preparing forms, and this is never entirely absent, even in structures in which the greatest possible use is made of steel forms and repetition details.

Against this, reinforced brickwork of the modern type, to which the author calls attention, requires little falsework beyond strutting and lagging of soffits or centers for arches. The main structural material, however—the brick—must be placed entirely by hand labor of an expensive kind. The brickwork structure, too, must be built with a vast number of joints, the positions and arrangements of which are determined beforehand by the dimensions of the brick, the system of bonding, and the shape of the completed structure. There is, therefore, much more limitation upon the disposition and permissible shape of the reinforcement than there is in the equivalent concrete structure.

These general considerations show that both systems of construction must have their spheres of usefulness among the range of practical conditions under which structures must be built. The development of reinforced brickwork was largely arrested during the second half of the Nineteenth Century because of lack of scientific knowledge of design, and during the earlier years of the Twentieth Century by the continuous improvement in the manufacture of cement. The technique of preparing concreting materials, of making concrete, and of scientific designing of concrete structures, has been greatly advanced. Now there is no reason why reinforced brickwork should not take a recognized place among structural materials. In the days of the Brunels and the other pioneers of reinforced brickwork, hand-made bricks only were available. The modern machine-made brick is a far superior product in strength, shape, and uniformity of quality; it conforms to much closer limits of dimension and is in every way a more dependable product. It

remains only to put the design of reinforced brickwork on a sure basis, so that engineers may have available an alternative structural material, which they will be able to use with advantage when economic conditions are suitable.

All those who have experimented with reinforced brickwork are agreed that the same fundamental bases of design may be used as are generally accepted for reinforced concrete. There is no reason for disputing this conclusion, since the fundamental conditions must be the same in both cases. The flexibility, which is provided in a reinforced concrete beam by the fact that the concrete will resist small tensile stresses and thus accommodate itself to the development of larger tensile stresses in the reinforcement, is provided in the brick beam by the mortar of the joints. Concrete and brickwork are equally suitable for compressive loading. Conditions for the development of bond stresses should be the same in brickwork as in concrete if adequate sectional area of mortar is provided around the reinforcement and if attention is paid to the proper filling of all joints in which steel is embedded. Provision of shear reinforcement is perhaps more difficult, since in its arrangement attention must be given to the disposition of the individual bricks. The limitations that result from this may also curtail the freedom of the designer to use the reinforcement to the greatest advantage.

No mention is made in the paper of the effect of brick bond on the strength of reinforced brickwork. It seems reasonable to suppose that variations in bonding must have their effects on strength, particularly on shear strength. This is a feature in which direct comparison is not possible with reinforced concrete, where for design purposes the concrete section may be taken as uniform. Vertical sections of a brick beam change suddenly in character, according as they are taken through vertical joints or not, and the arrangements and disposition of the vertical mortar joints must vary according to the bond adopted in building the beam. It seems likely that this variability of conditions has its effect on the distribution of shearing resistance and may account in some measure for the large variations of unit bond and shear stresses shown in the author's Table 2. The writer would like to see this point investigated more fully.

The summarization of experimental results given in the paper shows that there are very wide ranges of ultimate strength. On the whole, these ranges are much wider than are found in modern researches on reinforced concrete. While it is possible to take from them reasonable figures to form a basis of design, it certainly seems that considerable further co-ordinated research is necessary in order to put design on a sure and economic basis. Mr. Hansen mentions cases in which the full value of experimental results cannot be attained, because of uncertainties in the conditions under which the observations were made. In reinforced brickwork the distributions of stresses are probably more complicated than they are in reinforced concrete. When the volumes of research on the two materials are compared, one is driven to the conclusion that there is a large field still open for co-ordinated investigation before generally accepted rules of design for reinforced brickwork can be laid down with the degree of certainty necessary for its full development as a structural material. The writer hopes that one of the results of the presentation of this paper may be that such research will be undertaken.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

HIGH DAMS ON PERVIOUS GLACIAL DRIFT

Discussion

BY MESSRS. JOEL D. JUSTIN, AND A. K. POLLOCK

JOEL D. JUSTIN,[11] M. AM. SOC. C. E. (by letter)[11a].—The old textbooks taught that earth dams should be built only of impervious materials on foundations of like materials, that masonry core-walls reaching to bed-rock were, to say the least, highly desirable, and that concrete spillways and power houses should never be built except on solid ledge rock.

This idea has become so thoroughly engrained in the minds of some engineers that they have refused to consider the feasibility of dam sites where such conditions were impracticable of attainment and, in other cases, have insisted on the expenditure of vast sums to secure such conditions. With regard to hydro-electric developments this attitude has been responsible for the rejection of projects that would have proved economic if designed and built on the principles followed in Michigan and described by Mr. Burd. In fact, the successful functioning of numerous hydraulic structures on pervious foundations in Michigan and elsewhere is proof positive that it is not necessary to have the ideal conditions specified by the old textbooks in order to construct successful dams of concrete or earth. As conclusively shown by Mr. Burd, it is merely necessary that all the pertinent conditions be analyzed and understood in order to be able to design and build successful hydraulic structures on pervious foundations.

The author states that "the advance of steam generation has reduced the hydro-electric field to that of supplying peak power." Throughout considerable sections of the United States this statement is measurably correct. However, the restriction is a broad one and leaves ample room for economic hydro-electric development to an extent exceeding past experience. Statistics show that the utilization of water power relative to steam is not decreasing but that it has been increasing in the past few years. In the year ending June 30, 1933, 41.4% of all the electrical energy generated in central stations of the United States was from hydro-electric plants, as compared to

NOTE.—The paper by Edward M. Burd, M. Am. Soc. C. E., was published in April, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: May, 1933, by Messrs. Charles W. Sherman, and F. B. Marsh; and September, 1933, by M. M. O'Shaughnessy and H. de B. Parsons.

[11] Cons. Engr., Philadelphia, Pa.

[11a] Received by the Secretary September 7, 1933.

35.9% for the previous year. In 1927, of all installation in central stations of the United States, 26.0% was hydro-electric and, in 1931, this had increased to 28.3 per cent. The average annual capacity factor for all central station steam plants in the United States for the year ending June 30, 1933, was 21.1%; whereas, the corresponding value for central station hydro-electric plants was 39.8 per cent. These facts are interesting, particularly as they are quite contrary to what is frequently assumed to be the case.

The frequent superiority of hydro-electric power for peak-load service is just beginning to be recognized by public utility executives.[12] Primarily, this superiority arises from the fact that the unit incremental cost of hydro-electric installation is low; that is, the cost per kilowat of installation of intake, conduit, power house, and hydraulic and electrical equipment, etc. In many cases, such incremental cost for hydro-electric projects with ample weekly pondage varies from $50 to $75 per kw. If the relation between minimum stream flow at the time of system peak load and the system peak load is such as to permit such hydro-electric capacity to operate when required, it performs the same service as that done by the ancient steam plants which are retained in service for the purpose of supplying the upper portion of the annual peak load.

The ancient steam plant requires a considerable crew to maintain it and keep it ready to operate when required and even if no useful energy is produced, a considerable quantity of coal must be burned. The annual unit cost of such old steam plants, excluding any return on the investment, varies from $8 to $15 per kw per yr, without the generation of any useful energy. On the other hand, maintenance and operating costs on the alternative hydro-electric units are insignificant and the total annual cost, including fixed charges on the investment, may be from $5 to $8 per kw per yr on an incremental basis.

In any proposed plan for superseding old steam capacity by hydro-electric units, it is essential, of course, first to make sure that the hydro-electric units can perform the same functions as those required of the superseded steam plant. Hydro-electric plants with ample reservoirs and pumped storage hydro-electric plants are both promising possibilities for peak load and reserve service.

Mr. Burd states that, with the embankment materials utilized in such structures as the Hardy Dam in Michigan, a percolation factor of 5 is desirable. This means that the shortest distance that water could travel from the reservoir to tail-water, or to the down-stream side of the embankment, should be not less than five times the head. He then proceeds to state that for finer materials the percolation factor should be greater. In other words, the slope of the line of saturation would be flatter if finer materials were used. Such a statement requires qualification as it is not necessarily true. As has been pointed out by the writer,[13] the slope of the line of saturation is determined not only by the size of the particles composing the embankment but also by

[12] This subject is treated at some length in "Power Supply Economics," by Joel D. Justin and W. G. Mervine, John Wiley & Sons, Inc., 1933.

[13] "Earth Dam Projects," by Joel D. Justin, John Wiley & Sons, Inc.

(1) the particles composing the foundation material; (2) the porosity of both foundation and embankment; (3) depth below base of dam to rock or other impervious material; (4) ground-water conditions; (5) temperature; and, last but not least, (6) provisions made for drainage.

Consider a dam built of materials finer than those used in the Hardy Dam and in which the materials in the foundation have the same degree of fineness as those used in the embankment, and the same porosity; and that the natural ground-water is at the same elevation as it was at the site of the Hardy Dam. Then, the line of saturation would be at a slope much flatter than that which occurs with the coarser materials at the Hardy Dam site. The percolation factor would thus be higher. This is in accord with the statement made by the author.

On the other hand, if the embankment were built of much finer materials than those used at the Hardy Dam, but the foundation material was coarse, the slope of the line of saturation in the dam might be much steeper than was the case at the Hardy Dam and, consequently, the percolation factor for the dam would be less. Artificial drainage in a dam of finer material might also cause the percolation factor to be smaller.

Fine embankment materials do not necessarily require flatter slopes than those used at the Hardy Dam for coarse materials. Thus, many successful dams of clayish materials, built in rolled layers, have slopes as steep as those utilized at the Hardy Dam.

Many construction superintendents dislike a core-wall extremely. It usually interferes with the operations of embankment placing and it is claimed that it increases the total cost of the work to a greater extent than can be shown by any cost segregation. Apparently, the cost of the core-wall at the Hardy Dam was only about 15% of the total cost of the embankment. In some cases with which the writer is familiar, the cost of the core-wall, including the cut-off and excavation therefor, has been as much as 50% of the total cost of the embankment. The advisability of using a core-wall and the steel sheet-piling under it is largely a matter of economics. It has been the writer's experience that where a cut-off can readily reach ledge rock or an impervious layer, and where only relatively pervious material is available for the embankment, a core-wall of the Michigan type is frequently economical. Where there is no impervious layer to be reached, and where embankment and foundation materials are homogeneous, the writer has usually found it more economical to omit the core-wall and secure the necessary percolation distance by flattening the slopes of the dam, sometimes with the added precaution of an up-stream blanket of fine material.

From a theoretical standpoint, the idea of placing an impervious membrane on the up-stream face of an earth dam instead of using a core-wall has all the advantages which the author claims for it. Quite a few dams have been built with such a membrane, but the results have not always been satisfactory. A reinforced concrete core-wall, such as those commonly used in Michigan and elsewhere, is buried in the body of the dam and protected from the action of the elements. It is subject to only slight distortion, which

it may be designed to take without losing its impervious character. If designed and constructed with moderate skill, it may be relied on to retain its relatively water-tight characteristics.

One cannot feel the same assurance with regard to a reinforced concrete diaphragm laid down on the inclined up-stream face of an earth dam. Settlement may cause distortion and, with the aid of temperature changes, cracking of the diaphragm; or, if concrete blocks are used, these may be displaced, opening cracks at the joints. In case the water in the reservoir is drawn down suddenly internal hydrostatic pressure in the embankment may cause rupture of the diaphragm. A relatively small crack or opening through the concrete will supply a very large area of the up-stream face of the embankment with as much water as it could absorb if entirely unprotected by the concrete diaphragm.

On the Owl Creek Dam, near Belle Fourche, S. Dak., during a period of high waves, the receding waves momentarily relieved the pressure on the concrete facing blocks, so that the internal hydrostatic pressure from the embankment forced out about two hundred and fifty of them.

In the construction of rock-fill dams, reinforced concrete diaphragms on the up-stream face, usually of flexible jointed blocks, are standard practice; but, here, the backing is 100% free draining and in case of a rapid drawdown of the reservoir, there is no chance of an unbalanced upward pressure.

Even with the sand dams of Michigan, it is conceivable that in case of a rapid drawdown of the reservoir, the water in the dam might be at a higher level than the water in the reservoir. The difficulty might be overcome by using a heavy fill of coarse material and rock on top of the inclined reinforced concrete paving. The load placed on the paving should be great enough to counterbalance any upward hydrostatic pressure which might come on the bottom of the concrete slab as a result of a rapid drawdown. To insure that the fill on top of the concrete was not displaced by wave action, it would have to be protected by rip-rap or large stone. In many cases, such requirements would take away all the savings claimed for the plan.

Concrete lining has been used to a large extent on the water slopes of the distribution reservoirs of water-supply systems. The function in this case, however, is usually not to secure water-tightness, but to obtain a smooth hard surface to facilitate cleaning and also to protect the embankment against wave action. Concrete blocks, often laid on a bed of gravel, also have been used frequently on the up-stream face of high earth dams to provide protection against waves. In such cases it has usually been found desirable to provide weep-holes or other spaces through the concrete blocks so that when the water in the dam is at a higher level than the water in the reservoir, it can seep out without causing an unbalanced hydrostatic pressure on the bottom of the concrete blocks.

A. K. Pollock,[14] Assoc. M. Am. Soc. C. E. (by letter)[14a].—The work described in this excellent paper is perhaps rather bold and daring in its

[14] Asst. Res. Engr., Barnsley Corporation Water-Works, Barnsley, Yorkshire, England.

[14a] Received by the Secretary September 25, 1933.

conception, but the result would appear to have justified the designer's optimism. It is not novel to build a dam on a sand foundation, but the height of the Hardy Dam makes it a notable structure, especially when it is remembered that the dam itself is also of sand.

Bligh has given[15] comprehensive data on the construction of weirs on sand foundations, and Strange developed[16] a type of earth dam for use where percolation under and through the dam was unavoidable. The chief feature of this type is the extensive system of under-drainage.

The percolation factor of 4.25 used at the Hardy Dam is lower than any advocated by Bligh, who gives a factor of 12 for coarse sand and a range of 9 to 5 for shingle, gravel, and sand mixed. Mr. Burd states that "experience has indicated a safe generalization for this form and composition of embankment, namely, this plane [of saturation] remains below a 1 on 5 slope from head-water to tail-water." From Fig. 5 the material at the Hardy Dam would be classed as coarse sand. The author's experience, therefore, would seem to indicate that Bligh's factors are rather on the conservative side.

A curious feature of the Hardy Dam and others of its type, is the practical cessation of settlement on the completion of construction. Sand is virtually incompressible, of course, and most of the settlement that takes place is due no doubt to the packing and arranging of the grains of sand during the washing-in process. A point not mentioned in the paper, is the effect on the dam of the bulking property of sand. It would be interesting if the author could state what, if any, were the observed effects of the decreasing moisture content of the dam as the water used in construction drained away. Experiments in connection with the measurement of sand for batching concrete have shown that saturated sand has approximately the same volume as dry sand, but that when the moisture content is in the order of, say, 5 to 10%, the sand increases in volume perhaps as much as 20 per cent.

During the construction period of the Hardy Dam, it is assumed that the water used for washing the sand into place was sufficient to saturate the material which would then occupy its minimum volume. As the water drained away, the percentage moisture content must have dropped to a point at which measurable bulking would occur. How did this affect the settlement records? When the reservoir was filled, that part of the embankment below the line of saturation would revert to the condition of minimum volume, but the part above the line of saturation would probably reach a condition where the moisture content was such as to cause maximum "bulking," and this part of the bank, therefore, would be in a condition of maximum volume. Observations of settlement on stakes placed on the downstream slope would be difficult of correct interpretation if this "bulking" did occur. If there was no real settlement a stake would rise with the bulking of the embankment. If the peg showed no movement, may not this be interpreted to mean that the bulking of the material counteracted the settlement?

[15] "Dams and Weirs," by W. G. Bligh, 1927.

[16] "Indian Storage Reservoirs with Earthen Dams," by W. L. Strange, Third Edition, 1928.

The foundation settlement appears to have been observed only by reference points on the core-wall, power house, and penstocks. Of course, this does not give the settlement of the valley bottom due to the superimposed weight of the dam, and it would be interesting if Mr. Burd could give any information on this point. Due to the core-wall being founded on sheet-piling, there would be a considerable differential settlement between the core-wall and the valley floor, on either side of it.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ACTUAL DEFLECTIONS AND TEMPERATURES IN A TRIAL LOAD ARCH DAM

Discussion

BY MESSRS. D. C. HENNY, AND B. E. TORPEN

D. C. HENNY[8], M. AM. SOC. C. E. (by letter)[8a].—The clear and complete manner in which the results of research at Ariel Dam have been presented does great credit to the authors. The very thoroughness of the paper renders correction of any misstatements all the more desirable. Attention is called, therefore, to what was said as to sprinkling of the concrete, as follows, "* * * all surfaces of the arch, as well as those of the remainder of the dam, were sprinkled continuously with water" and again, elsewhere, "the outside surfaces exposed to the sun were kept wet constantly by sprinkling." These statements are correct only as to the advice given by the writer in his capacity of Consulting Engineer of both the State and the Northwestern Electric Company. He deemed undisturbed continuation of the setting process, early cooling, and prevention of shrinkage from drying for a period of at least two months, of great importance, especially in the present instance where the heat rise in the concrete was rather high (70°F) and where closure of the arch had to await cooling of the concrete. The facts, however, were that for about ten days the concrete surfaces were kept moist most of the time and thereafter they were sprinkled occasionally, although somewhat better results were obtained during the latter half of the work.

In a number of arch blocks slight checks and cracks developed where the dimensions were about 30 and 40 ft one way and 40 to 60 ft the other way. They occurred on all faces generally half-way between corners and were due apparently to continued heat rise in the interior and to simultaneous shrinkage of the outside resulting from drying and cooling. Investigation by hydraulic test showed that these fine cracks did not penetrate more than 3 or 4 ft and, in the course of time, their width diminished. On later examination

NOTE.—This paper by A. T. Larned and W. S. Merrill, Members, Am. Soc. C. E., was published in May, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1933, by George Jacob Davis, Jr., M. Am. Soc. C. E.

[8] Cons. Engr., Portland, Ore.

[8a] Received by the Secretary August 28, 1933.

these cracks seem to be entirely closed, and where moisture conditions were favorable they may, in fact, have become completely healed.

In the matter of uplift it was stated that "the maximum observed uplift between the rock and the base of the dam was less than 20% of the static head and this was within 5 ft of the up-stream face." An important question is to what extent this favorable condition was due to foundation drainage. On the whole it is believed that the job of securing a good foundation and of grouting was done so thoroughly that even without drainage the uplift pressure would have been relatively low. For a part of the work, however, this is not true. Reference is made to the abutment block next to the arch on the north side of the river which is shown in plan in Figs. 4 and 6. From these diagrams it may be noted that this block has an addition projecting into the reservoir, which was made necessary by the heavy radial shear transmitted by the arch. A branch of the main drainage gallery extends into this addition, in which six vertical 2-in. pipes rise from the foundation. They are about 10 ft from the up-stream face and their aggregate flow is approximately $\frac{2}{3}$ gal per min with reservoir full and the water level 130 ft above the top of the pipes. With reservoir water 40 ft above the top of the pipes during the period of first filling, a test was made by closing all the pipes when it was found that the pressure quickly rose to 80% of the reservoir head.

The block projection in the gallery in which this pressure was observed, is exposed to increep of water at the foundation line on three of the four sides, which implies a favorable condition for the development of high uplift pressure. This experience illustrates two points, namely, that on the one hand thorough drainage is desirable and on the other that no absolute dependence should be placed on it in the design.

Mention was made in the paper of the method of making vertical cooling holes with rubber cores. These cores are quite effective in mass concrete and have the great advantage of keeping the holes clean and free of trash. However, it was found difficult to maintain the holes straight and in their exact position. When an attempt was made to use them in the 2 ft of slot-fill concrete great difficulty was found in pulling them up without disturbing adjacent concrete, and their use in this connection was abandoned. Incidentally, it may be said that the only seepage now (1933) visible on the downstream face of the dam (except that coming through three small diagonal cracks from 3 to 5 ft long near the abutments), occurs through the concrete slot fill. This seepage is small and consists mostly of discoloration of the concrete.

It was to avoid such seepage that artificial cooling in connection with reasonably slow construction was urged. Seepage of this kind had been observed in similar cases; it is evidently due to the fact that vertical contraction during cooling is prevented by friction against the sides of the old concrete thus forcing the formation of fine horizontal cracks where, at construction joints or other places, tension exceeds the tensile strength.

As regards both deflection and temperature measurements new data are constantly becoming available at this structure, and it is hoped that the authors in their closing discussion will include this additional information.

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The block projection in the gallery in which this pressure was observed, is exposed to increep of water at the foundation line on three of the four sides, which implies a favorable condition for the development of high uplift pressure. This experience illustrates two points, namely, that on the one hand thorough drainage is desirable and on the other that no absolute dependence should be placed on it in the design.

Mention was made in the paper of the method of making vertical cooling holes with rubber cores. These cores are quite effective in mass concrete and have the great advantage of keeping the holes clean and free of trash. However, it was found difficult to maintain the holes straight and in their exact position. When an attempt was made to use them in the 2 ft of slot-fill concrete great difficulty was found in pulling them up without disturbing adjacent concrete, and their use in this connection was abandoned. Incidentally, it may be said that the only seepage now (1933) visible on the down-stream face of the dam (except that coming through three small diagonal cracks from 3 to 5 ft long near the abutments), occurs through the concrete slot fill. This seepage is small and consists mostly of discoloration of the concrete.

It was to avoid such seepage that artificial cooling in connection with reasonably slow construction was urged. Seepage of this kind had been observed in similar cases; it is evidently due to the fact that vertical contraction during cooling is prevented by friction against the sides of the old concrete thus forcing the formation of fine horizontal cracks where, at construction joints or other places, tension exceeds the tensile strength.

As regards both deflection and temperature measurements new data are constantly becoming available at this structure, and it is hoped that the authors in their closing discussion will include this additional information.

The paper gives well deserved credit to the Company engineers and superintendents, but inadvertently omits mention of D. W. Cole, M. Am. Soc. C. E., who was responsible for securing proper foundation, grouting, and many other important features of the work.

B. E. Torpen[9], M. Am. Soc. C. E. (by letter)[9a].—The authors are to be commended for their full presentation of the data on dam deflections and concrete temperatures. It is hoped that follow-up measurements will be made and published at intervals of a year or two. By co-operative work of this nature at several dams, their behavior will become known in time, and such information will be useful in future design.

As yet little is known of the internal behavior and stresses in dams. Research and tests are very necessary and each additional ray of light tends

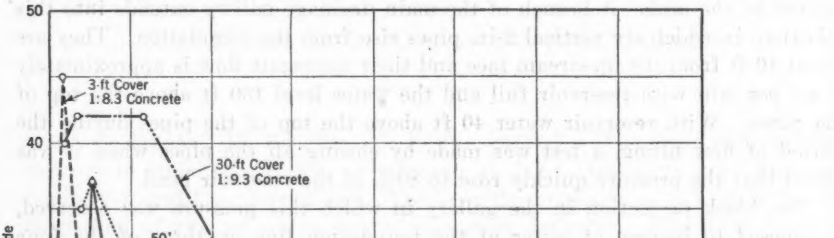

Fig. 25.—Concrete Temperatures, Bull Run Dam, Portland, Ore.

to brighten the hope that some day dams may be designed with the assurance and the knowledge that they will serve as planned. To-day, there is too much uncertainty to permit unity of opinion among designers. It will require time, tests, research, and data on existing dams to crystallize opinion

[9] Portland, Ore.

[9a] Received by the Secretary August 28, 1933.

into a common knowledge. In addition to recording deflections, every effort should be made to determine the internal stresses in the concrete due to water load, heat, water-soaking, and the time-factor influence.

The question of temperatures in setting concrete has long been of interest to engineers, and temperature readings have been observed since 1910, or thereabouts. At the Bull Run Dam, for Portland, Ore., fifty-two resistance thermometers and telemeters were buried in the mass of four 40-ft sections at various depths, and temperature readings were observed for several years. The results of these tests have been compiled[10] by Ben S. Morrow, Assoc. M. Am. Soc. C. E.

Temperatures at Bull Run Dam shown in Fig. 25 indicate that at 3 ft of cover, the concrete soon loses its chemical heat and follows the air temperature with a lag of about 5°C and one or two months in time. At 8.5 ft of cover, the cooling is slower and the effect of air temperature is less, but still quite marked. At 30 ft of cover, cooling is very slow, requiring about three years to return to the temperature at which it was placed.

The curve at 50 ft of cover is influenced by the proximity of foundation rock, and all curves are affected by water-soaking from the up-stream face, as the reservoir was allowed to fill in May, 1929. The "average" curve shows the influence of the fluctuating temperature of the exterior envelope.

At four years, the average annual range for the entire dam was only 5°C; at 3 ft of cover, it was 10°C, or about one-half the annual range of the air, which is 18°C.

The specific gravity of the aggregate at Ariel Dam was 2.38 as compared to 2.8 for aggregate at Bull Run (see Table 9).

TABLE 9.—COMPARISON OF AGGREGATE AND RESULTING CONCRETE

Item	Ariel Dam	Bull Run Dam
Specific gravity, gravel	2.38	2.8
Weight of concrete, dry, in pounds per cubic foot	138	154
Weight of concrete, wet, in pounds per cubic foot	147.5	157.9
Cement content, in barrels per cubic yard	1.00	0.88
Water cement ratio	1.05	0.97
Slump, in inches	1	3 to 4
Breaking Strength, in Pounds per Square Inch, at:		
28 days	2 800	2 100
6 months	3 750	.....
1 year	.....	3 500

Arch Site.—The Ariel Dam site does not appear especially favorable for an arch, but careful comparison of other designs showed this type to be the most economical in spite of the 2.3 to 1 ratio of span to height for the arch alone, and 3 to 1 ratio of span to height when the entire length of the dam site is considered. The variable radius feature of this arch dam has been in use only since 1912, but, since then, about fifty dams of this type have been constructed. As more data become available on arch deflections and stresses, this type of dam will become even more popular, due to its economy.

The authors have presented a mass of important data. The Ariel Dam is a beautiful structure, well built, and a fine example of modern engineering.

[10] Rept. of the Committee of Engineering Foundation on Arch Dam Investigation, May, 1933, Vol. III, Pt. III, p. 41.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

EARTHS AND FOUNDATIONS PROGRESS REPORT OF SPECIAL COMMITTEE

Discussion

BY MESSRS. EDWIN J. BEUGLER, JACOB FELD, GEORGE D. CAMP, AND CHARLES TERZAGHI

EDWIN J. BEUGLER,[36] M. AM. SOC. C. E. (by letter).[36a]—The Committee is to be commended for this report, which is a valuable addition to the good work of former committees. The report incidentally contains a number of debatable points which should promote active and healthy discussion. Some criticism may be made on the score of too much abstract science and laboratory work and not enough consideration of field experience and the application of judgment. Scientific investigation, however, is essential to formulating physical laws which when soundly developed may be confidentially applied to the design of new foundations or to the remedy of defective ones.

In connection with the distribution of single or multiple loads, it is interesting to note the harmony between results of the tentative present-day formulas and that of Boussinesq, and the striking confirmation by use of photo-elastic experiments of the so-called "pressure bulb" action as set forth several decades ago.[37]

The assumptions of completely saturated clay and settlement caused by the load squeezing out the moisture, are justified for a simple and clear preliminary study of the phenomena connected with these particular conditions. Further study may then be made under the assumption that a part of the settlement takes place by reason of plastic flow laterally or upward. This particular phase may explain several notable foundation break-downs, among which was the settling and tilting of a large grain elevator near Winnipeg, Man., Canada.[38]

NOTE.—The Progress Report of the Special Committee on Earths and Foundations was presented at the Annual Meeting, New York, N. Y., January 18, 1933, and published in May, 1933 *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: August, 1933, by Messrs. L. C. Wilcoxen, H. de B. Parsons, William P. Kimball, and T. A. Middlebrooks; and September, 1933, by Messrs. Daniel E. Moran, and A. E. Cummings.

[36] Cons. Engr., Cheshire, Conn.

[36a] Received by the Secretary August 17, 1933.

[37] *Public Service Record*, N. Y., September, 1915.

[38] "The Failure and Righting of a Million-Bushel Grain Elevator," by Alexander Allaire, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 799.

The examples of settlement given in the report are all of interest. The first four are clearly examples of overloading, and most of the settlements, as indicated, are beyond the range of permissible movement in good structural design. With given loads, the design should provide for their distribution laterally and downward for adequate support without appreciable settlement under such loads. Some designers have increased bearing capacity by the removal of a certain amount of soil over-burden to offset a part of the structural load to be imposed. In this connection, a definition of the term, "settlement," should be clearly understood.

The study and application of soil mechanics require a differentiation between two important factors of the so-called settlement, as indicated in the examples referred to: First, compression taking place during the erection of the structure; and, second, what may be called "true settlement" under a definite uniform load, subject only to minor variations from live load and impact. The first factor has little to do with the bearing capacity of the soil; the second one is all-important and is a clear indication of supporting capacity. The example of the Washington Monument is a good illustration. It shows that when the monument was raised from a height of 150 ft to 500 ft, the settlement or compression during the application of the increased load to the time of completion amounted to 4.5 in. Subsequently, under the constant load of the completed structure for a period of nearly fifty years, the settlement was about 1 in.—a truly modest amount and a tribute to the engineers who designed the enlarged foundation at the time of increasing the height of the monument.

When making a skeleton analysis of a situation, such as that shown in Table 6, it would seem more logical to take the time as zero when the structure is completed; that is, when the total dead load has been applied. Subsequent movement will then be *bona fide* settlement as distinguished from compression while construction is in progress.

The prime purpose in the use of piles is to transfer the load economically to a lower stratum having greater bearing capacity. A case such as that shown in Fig. 31 indicates a serious lack of knowledge of the underground formation. There is no justification for forcing piles through 30 ft of sand and gravel only to end them immediately above a deposit of mud. With respect to piles driven through clay of considerable depth, two advantages may accrue over ordinary shallow footings without piles: First, the clay will be more or less consolidated by pile displacement, largely if not altogether offsetting any remoulded effect, besides materially increasing the lateral resistance of the clay. This has been found advantageous in connection with foundations for retaining walls and swing draw-bridge piers in soft ground. Then, again, the load may sometimes be transferred to a lower clay stratum of the same physical make-up but with greater density due to the weight of the superimposed clay.

In view of the apparent difference between the test results on undisturbed clays and remoulded samples shown in Fig. 19, together with the fact that footings without piles are generally placed on the surface of undisturbed

material, it is suggested that bearing tests might well be confined to the undisturbed clay *in situ* rather than make tests on small samples requiring extreme care in order to maintain them in their natural undisturbed state. Where the proposed footings will be above the water line, test pits may be sunk at low cost and definite tests made to determine the soil conditions at the surface on which the foundation is to rest. On the site of the Municipal Building, in New York City,[39] bearing tests were made through a 16-in. casing at different depths—some as much as 77 ft below curb grade—on a circular disk having an area of 1 sq ft, with unit loads as great as 25 tons. Obviously, for all important structures core borings are necessary to determine the character of the material beneath them. Wash borings may be very misleading unless supplemented by examination of undisturbed soil samples.

Aside from inexperience and lack of judgment, the endeavor to load the foundation bed to the extreme limit has been the cause of most foundation failures, as far as settlement is concerned. An ideal design would produce a structure in which no further settlement takes place after the natural compression due to the initial application of the load during construction or shortly thereafter. In many cases no uneven settlement is permissible. An illustration is the case of the foundations of the Municipal Building, New York City. Under two-thirds of this structure the footings were carried to solid rock, and under the other one-third it was founded on sand. Stability was assured by using a unit load of 15 tons for the rock section and a reduced unit load of 6 tons for the sand, although tests showed no settlement after initial compression under a 10-ton load on the sand. In studying actual examples of various foundation methods and results, particular attention should be given to structures built in difficult places which have not settled, rather than examples of questionable design where settlements have reached several feet.

The effect of surcharge on bearing capacity of soils is another important item of soil mechanics. From a practical standpoint, it is desirable that general laws be developed for the action of soils under footings placed at various depths under a given surface. The various phases, including lateral soil resistance, density increasing with weight of surcharge, etc., are so related that the composite problem is not a simple one.

There is more or less uncertainty as to the influence of the area of footings on the relative unit bearing capacity. Conversely, this phase is important in determining the variable unit load to produce the same settlement with different sizes of footings or mat areas on a given soil. Experiments show that soils of a clay type may carry less unit working load as the area increases, while non-cohesive materials may carry a greater unit load with the increased area. For example, the grain elevator settlement noted previously took place under a large concrete mat on blue clay with a unit load of about $2\frac{1}{2}$ tons per sq ft, while small footings of a coal trestle near-by actually carried twice the unit load without evidence of any settlement.

[39] *Engineering News*, November 17, 1910.

Pending the further development of soil mechanics applicable to this matter, the writer has suggested a simple method for the practical determination of working values for bearing capacity of footings of different sizes, based on field tests with two or three areas ranging, say, from 1 sq ft to 9 sq ft on the specific soil in question.[40]

It is hoped that the Committee will continue its labors, developing the different phases separately; then by combinations accurately build up a scientific backing for observed phenomena and incidentally formulate general rules and applied mechanics for the use of engineers having the responsibility of designing foundations for various types of structures on a still greater variety of soils.

JACOB FELD,[41] ASSOC. M. AM. SOC. C. E. (by letter).[41a]—The explanation of stress or unit area-support distribution in indefinite volumes, for the two cases of point and strip loadings, together with the limitations on a method of summation for other loadings, should clarify the conception of the "bulb of pressure" in the mind of the average engineer. It should be pointed out that the basis of the formulas is the assumption of a continuous isotropic medium of distribution. It is this assumption that explains the statement under the heading, "Pressure of Concentrated Loads": "The pressure distributions in both cases (which have been well substantiated by experiments), are independent of the type of material and involve no elastic constants." In a non-isotropic medium, this statement will not hold true.

As to the experiments, the loadings tested have been of such magnitude that the deviation from the isotropic condition did not seriously affect the results. It is to be noted that 10 tons per sq ft is equal to less than 140 lb per sq in. A load of such magnitude applied to a soil would show results differing from theory, but only within that volume of earth where the intensity of pressure exceeded the possible elastic tensile strains. All soils are elastic to some extent, and loads as great as 10 tons per sq ft are not usual. In isotropic strain, magnitude is independent of axes. In heterogeneous bodies, strains are not linear functions of stress and are not equal in all directions. However, if strains are small, the expansion of the strain function into a series of terms shows that the second and higher terms may be disregarded with only small errors. Hence, for small unit loads and for soils with some tensile value (cohesion), the assumption of isotropic strain is permissible.

Agreement of the theoretical computations with photo-elastic researches is to expected, since the elastic requirements are fulfilled. Agreement with field measurements of structures which have settled seriously, should not be expected.

Predicting settlements from laboratory tests of sub-soil samples will always remain a mixture of science and art. There is much of qualitative value in such estimates, but, if only for the reason that settlements are almost never

[40] *The Military Engineer*, July-August, 1933.

[41] Cons. Engr., New York, N. Y.

[41a] Received by the Secretary August 24, 1933.

uniform in any one structure even if founded on a so-called uniform soil, quantitative results must not be expected. If honest reason is taken into account in such estimates, there will be much less disagreement between the testimony of technical experts called in by opposing sides in a lawsuit concerning foundation settlements.

The progress report represents considerable work and the Committee is to be complimented on this contribution to the advance of knowledge on earths and their use as a structural foundation.

George D. Camp,[42] M. Am. Soc. C. E. (by letter)[42a].—The theory, as outlined in the report, appears reasonable and rational but, as it does not give results in accord with experience under certain conditions (notably those to be found in the City of Mexico), it would seem to be applicable rather to a special case than to a general one. While these local conditions (in Mexico) are strikingly different from those that have been studied most widely, it is believed that some of the principles most apparently involved here—although their application to conditions elsewhere is not very evident—may also apply generally to other conditions even if in a minor respect.

The "Basic Problem," as stated in the report, may be true enough for certain kinds of soil but the types to which it applies should be stated. To illustrate the inadequacy of the "problem" as stated, one example of an extreme nature will suffice. The site for a large factory was investigated for the purpose of foundation design. It was found that the first 10 or 15 ft from the surface was a fairly stiff, very cohesive, and rubbery clay beneath which the water content increased rapidly to as high as 95 per cent. The general characteristics of the soil were the same to a depth of 525 ft (the depth of a water well); that is, layers of soft clay with occasional very thin layers of sand and two or three layers, a yard or more thick, of coarse sand and gravel. Below the upper 10 or 15 ft, the various layers of clay had varying water contents, but they were all very high, ranging about 90%; and most of them were more truly fluid than solid. A hollow drill rod used for sampling sank of its own weight after going through the top crust, and the holes frequently closed up before the rods could be returned to them after cleaning. There was no distinct surface of contact between the upper crust and the lower strata, but the consistency shaded off gradually from a stiff rubbery material through thick, viscous clays to fluid clays, all in a depth of less than 33 ft from the surface.

Under these conditions, does a load, applied at the ground surface, distribute itself (see "The Basic Problem") "throughout the ground in accordance with fairly simple laws"? Is it true (see "Pressure of Concentrated Loads") that "the pressure distributions * * * are independent of the type of material and involve no elastic constants"? Long experience with the particular kind of soil described, on which most of the City of Mexico is built, has convinced the writer that settlement does not take place wholly "through the consolidation of the clay mass as the water is squeezed out

[42] Cons., Engr., City of Mexico, Mexico.

[42a] Received by the Secretary August 28, 1933.

under an applied loading," but that a great part of it is caused by the lateral flow of the viscous, fluid material from beneath the load.

The writer agrees with Professor Gilboy (see "Analysis of Clay") that "the properties of the material depend to a considerable degree upon the structure formed in Nature during the process of deposition and subsequent consolidation"; but how much subsequent consolidation is necessary before the theory as given in the report will apply? In a case such as that cited herein, where, geologically, consolidation seems to have scarcely begun, it appears reasonable to suppose that the clay may function as an elastic material on the first application of a load but that, as settlement increases, the structure breaks down and in its remoulded or puddled condition it no longer acts as an elastic material, nor even as a pyramid of uniform balls, but rather as a viscous fluid. Naturally, this change from an elastic material to a viscous fluid takes place slowly, the rate depending upon many factors such as: (*a*) The degree of natural consolidation reached; (*b*) the rate of application, the distribution, the intensity, and the amount, of the load; (*c*) whether the load is increased progressively or increased and decreased alternately; (*d*) the existence of vibration and impact; and (*e*) movement of the load, as in earthquakes.

The predominating clay of the basin of Mexico, to which the writer has given the name "texconite," exhibits the same characteristics as the Laurentian clay mentioned in the report, but to a much more extreme degree. As indicated in the report, phenomena of a similar nature to those found in texconite, may be found to some extent in all clay deposits. If this is true, it would seem that a theory for predicting settlements of structures founded upon clay should take into account these phenomena, and it is hoped that those engaged in research on this important subject will extend their studies to include such materials as exhibit these striking characteristics.

CHARLES TERZAGHI[43], M. AM. SOC. C. E. (by letter)[43a].—The most important part of the report is that which deals with the settlement of existing structures. In spite of the fact that the systematic observation of settlements represents, with few exceptions, a rather recent practice, it has already furnished contributions of outstanding importance to the knowledge of foundation behavior. The numerous examples cited in the report show, in agreement with general experience, that the settlement of structures is never uniform. In most cases the central part of the structure settles more than the outer parts, but there are many exceptions. In order to get a clear conception of the differential settlement, it is necessary to establish at least one bench-mark for every 80 sq ft of the base of the building. The levels should be made immediately after the foundations are constructed, once or twice during construction, at the end of the construction, and once every few months after construction is finished, until no more movements can be detected.

[43] Dr.-Ing.; Prof., Technische Hochschule, Vienna, Austria.

[43a] Received by the Secretary July 5, 1933.

Concerning the relation between load, time, and settlement, four principal types can be distinguished. They are shown in Fig. 54. Type (*a*) is confined to foundations on permeable strata (sand or gravel), and Types (*b*) to (*d*) to foundations on clay or mud. If a building rests on a thick layer of sand, underlaid by a stratum of clay, and it shows a time rate of settlement according to Fig. 54(*b*) to Fig. 54(*d*), it is certain that the stratum of sand

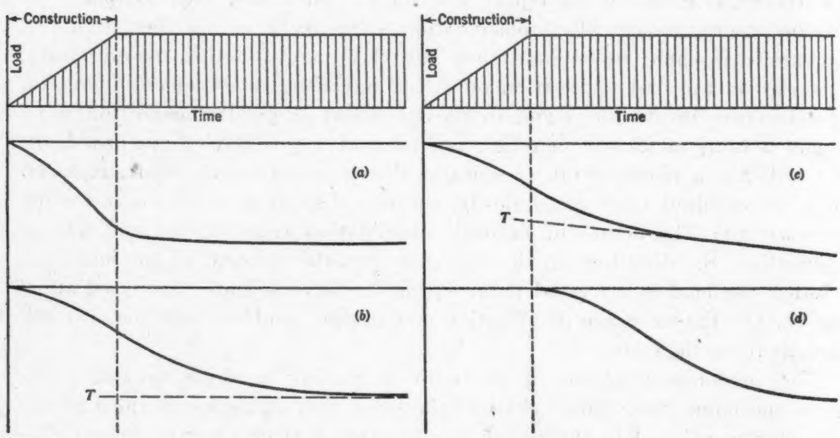

Fig. 54

subsides at the same rate as the building. For Type (*b*) the time rate of settlement gradually decreases, while for the Type (*c*), it seems to approach a fairly constant value. Thus, the building for which a time-settlement diagram is shown in Fig. 30 of the report, settles during the last 15 yr at a constant rate of $\frac{1}{3}$ in. per yr, and the Washington Monument (Fig. 34), at a constant rate of 0.023 in. per yr during the last 34 yr. The type represented by Fig. 54(*d*) is not yet explained and seems to be almost incomprehensible. Several cases of this type are already on record, and undoubtedly their number would increase if settlement observations became a general practice. In exceptional cases it happens that different parts of the same building settle in different ways despite the fact that the test borings may not have indicated a difference in the nature of the soil. Fig. 55 shows the settlement of two different points on a building which rests on conical piles, 20 ft long. All the piles were driven through a layer of artificial fill into a layer of firm gravel. It is not impossible (see Curve *A*, Fig. 55) that, beneath the piles, the gravel contained a pocket of stiff clay which is responsible for the time-lag shown by the curve.

If the owner of the building keeps a careful record of the elevation of the bench-marks he can prove, without expert testimony, that the construction activities of his neighbor, or the vibrations produced by the machines in a newly established factory, have had a detrimental effect on his property. To produce the proof it is sufficient to re-assume the settlement observations. If he wants to increase the weight of his building or to replace it by a new

and heavier one, the effect of the increase in load can easily be predicted from the results of the preceding settlement observations. The accurate knowledge of the behavior of the subsoil under load also increases the value of the building lot in the case of a sale. If the building gradually develops cracks due to a settlement of the type represented by Fig. 54(*b*) to Fig. 54(*d*), it is

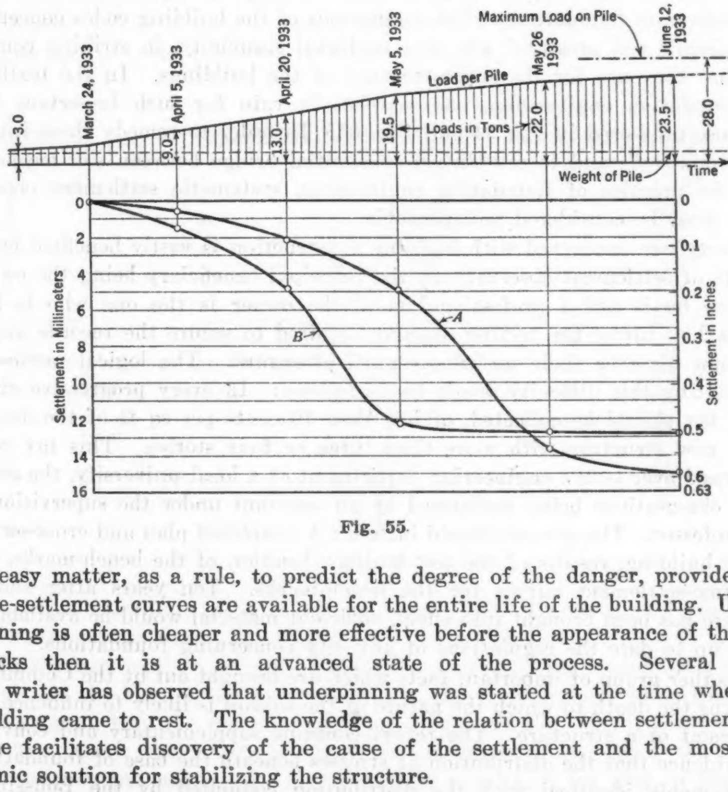

Fig. 55.

an easy matter, as a rule, to predict the degree of the danger, provided the time-settlement curves are available for the entire life of the building. Underpinning is often cheaper and more effective before the appearance of the first cracks then it is at an advanced state of the process. Several times the writer has observed that underpinning was started at the time when the building came to rest. The knowledge of the relation between settlement and time facilitates discovery of the cause of the settlement and the most economic solution for stabilizing the structure.

Compared to the aforementioned advantages which the owner derives from the results of settlement observations, the costs of keeping the records are negligible. Hence, it should be the duty of every conscientious consulting architect to include these observations in his work.

The contractor and the consulting engineer acquire, on the basis of settlement records, a clearer conception as to the maximum differential settlement which they can expect under different soil conditions. Suppose, for instance, one part of a proposed structure rests on a dense stratum of sand and the other part on a bed of stiff clay. If the contractor has already built on both the sand and the clay without making any settlement observations, he cannot even guess at the behavior of the foundation of the new structure one-half of which is on the sand and one-half on the clay. He merely knows that the previous building stood without cracking; yet, many buildings show no signs

of defective foundations in spite of an average settlement of several inches. Therefore, the differential settlement of the new structure could be of the same order of magnitude with the result that severe cracks appear above the boundary between the two subsoil materials.

Finally, the settlement observations are of outstanding economic importance. Every year important investments are lost or depreciated on account of inadequate foundations. The regulations of the building codes concerning "admissible soil pressure" are of a mediæval simplicity, in striking contrast to what they are for the superstructure of the buildings. In the textbooks on foundation engineering, one searches in vain for such important facts as those presented in Fig 54 and Fig. 55. In order to remedy these intolerable conditions and to establish a substantial bridge between soil mechanics and the practice of foundation engineering, systematic settlement observations must be considered indispensable.

Every one concerned with building construction is vastly benefited by the results of settlement observations, the principal beneficiary being the owner; because he is not a professional man, the owner is the one who is least inclined to invest the trifling amount required to secure the records and he does not discover their usefulness until afterward. The logical method of eliminating this difficulty would be as follows: In every progressive city a small tax should be collected, of less than 10 cents per sq ft of the area of every new structure with more than three or four stories. This tax could be turned over to the engineering department of a local university, the settlement observations being performed by an assistant under the supervision of the professor. The records should include: A simplified plan and cross-section of the building, results of the test borings, location of the bench-marks, and the time-settlement curves for the bench-marks. Ten years after such a measure has been brought into effect, sufficient material would be available to bring up to date the regulations of any city concerning foundations.

Another group of important facts which are brought out by the Committee concerns the depth to which the nature of the subsoil is likely to influence the settlement of a structure. The report contains supplementary and convincing evidence that the distribution of stresses beneath the base of foundations is reasonably identical with the distribution computed by the Boussinesq formulas. Due to the table (Table 1) devised by Professor Gilboy, the application of the formula has become extremely simple. For a homogeneous ground the actual soil pressures, beneath the center of the foundation, are somewhat greater than the computed values. If an elastic layer of soil rests on a more rigid stratum, the pressures are still higher than for a homogeneous subsoil. On the other hand, if the rigidity of the deeper strata is less than the rigidity of the upper layers, the pressures are less; yet, for practical purposes, one can assume that the computed values are correct.

From this well-established fact two conclusions of great practical importance can be drawn:

(1) The results of small-scale loading tests performed on the surface of the ground at the level of the base of a foundation can be utterly misleading

unless the subsoil is perfectly homogeneous to a depth at least equal to twice the width of the building.

(2) No reliable opinion can be expressed concerning the bearing capacity of a proposed foundation unless the nature of the subsoil is known to a depth at least equal to twice the width of the building.

Fig. 56 may serve to prove Conclusion (1). These three diagrams show the distribution of the soil pressures beneath the loaded area for a small-scale loading test (*a*); the base of a raft foundation (*b*); and the base of the foot-

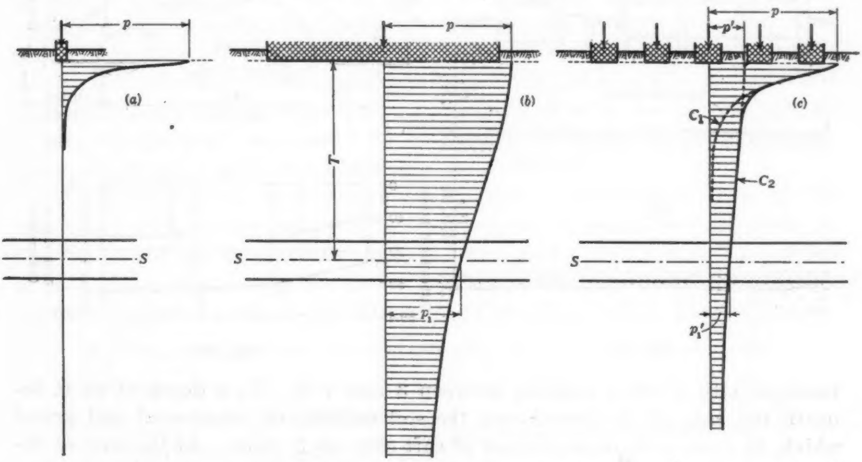

Fig. 56

ings of a building (*c*). When plotting these diagrams, the following assumptions were made: (1) The subsoil contains a layer, *S*, of soft material at an average depth, *t*, beneath the surface; and (2) the loaded area in Fig. 56(*a*) and the base of the footings in Fig. 56(*c*) are square, while the buildings supported by the foundations, Fig. 56(*b*) and Fig. 56(*c*), occupy a rectangular space, the width of which is considerably less than the length. The pressure per unit of the loaded area is equal to *p* for each of the three cases. Finally, it was assumed that the vertical distribution of the soil pressures is determined strictly by the Boussinesq formulas.

In the case of Fig. 56(*a*) the pressure which acts on the soft layer, *S*, is practically equal to zero. Hence, the occurrence of this layer has no influence on the results of the loading test. On the other hand, the settlements of the foundations in Fig. 56(*b*) and in Fig. 56(*c*) will be determined almost exclusively by the compressibility and the thickness of the stratum, *S*. The pressure diagram for Fig. 56(*c*) consists of two branches. The upper branch, C_1, represents the distribution of the pressures on the assumption that only the middle footing is loaded. The lower one, C_2, shows the same distribution under the assumption that the entire load is distributed uniformly over the area covered by the building. In the upper part, the pressure curve actually follows the branch, C_1; in the lower one, the branch, C_2. According to this

diagram, in spite of the small size of the individual footings, the building exerts an appreciable pressure on the soft stratum.

To illustrate the case, Fig. 56(*c*), the writer wishes to cite the settlement of the building in Fig. 57. The walls of this building rest on continuous

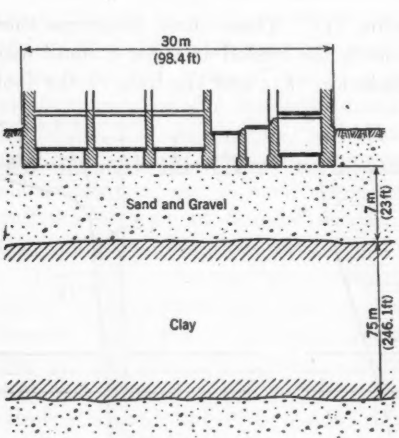

Fig. 57

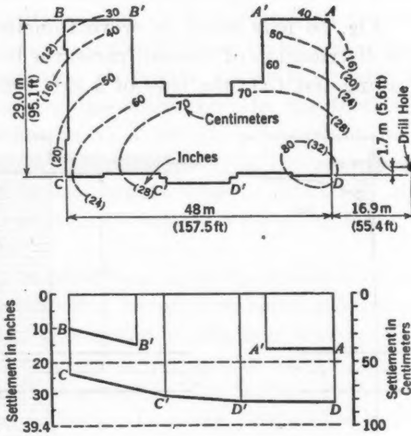

Fig. 58

footings with a width ranging between 3 and 4 ft. To a depth of 23 ft beneath the base of the foundation the soil consists of dense sand and gravel which, in turn, rests on a stratum of soft clay, 50 ft thick. At the base of the footings the soil pressure ranges between 3 and 4 tons per sq ft. In a loading test which was performed on an area of 1 sq ft, a soil pressure of 4 tons

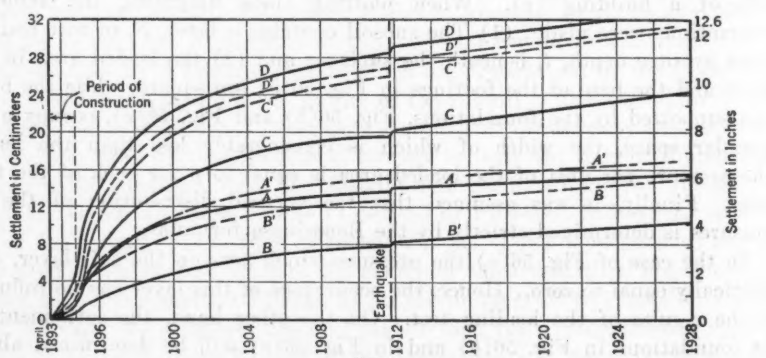

Fig. 59

produced a settlement equal to not more than a fraction of an inch. Nevertheless, during the forty years of its existence, the structure underwent unequal settlement, ranging between 12 and 32 in. (30 to 80 cm). Fig. 58 shows, the curves of equal settlement for 1928, and Fig. 59, the relation between time and settlement for the different bench-marks. Referring to Fig.

54, the settlement of this structure should be due entirely to movements within the clay in spite of the fact that the surface of the clay is at a depth of 23 ft beneath the base of the foundation. The unequal distribution of the settlement (Fig. 58) merely indicates that the softness of the clay increases toward the corner, *D*, of the building.

Plotting a pressure diagram of the type Fig. 56(*c*), one realizes that the surface of the clay lies far below the point of intersection of Curves C_1 and C_2 and that the pressure exerted by the weight of the building on the surface of the clay does not exceed 0.5 ton per sq ft. Due to the simplicity of the geological conditions, and to the reliable settlement observations covering a period of forty years, this case offered an unusual opportunity to compare the results of theoretical settlement computations with observed facts. For this reason the Committee appropriated the money required to make a test boring, to secure undisturbed samples of the clay and to determine all those soil constants which are needed for computing the settlement.

The investigations were carried out in 1931. According to Fig. 58 the drill hole is located approximately on the curve which passes through Point *A*. For computing the settlements a method was used similar to that described in the report. The results are shown in Fig. 60 (full-drawn curve), together with the settlement curve for Point *A* of the foundation (dotted curve).

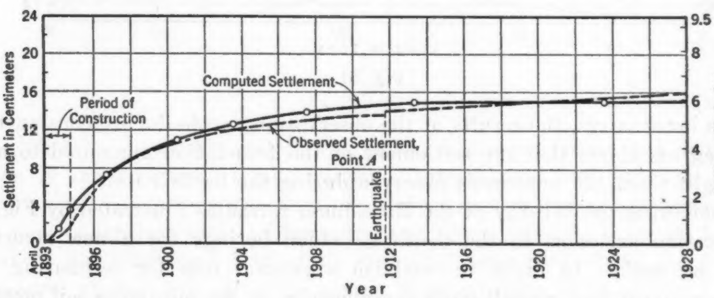

Fig. 60

For the first fifteen years of the process, the agreement between the two curves is so perfect that there can be no more doubt concerning the theory of the causes of the settlement being correct. However, for the later stage of the process, there is an important difference between theory and facts.

According to theory, the settlements should gradually stop while, in fact, they continue at an almost uniform rate of $\frac{1}{3}$ in. per yr. A satisfactory theoretical explanation has not been determined for this phenomenon. Therefore, one must find by experience which types of soils can be expected to continue settling, in contradiction to theory. The preceding example demonstrates both the usefulness of investigations of the mechanical properties of soil and the urgent necessity for systematic settlement observations. Only from a combination of both methods can satisfactory progress be expected. The example also demonstrates that no reliable conclusion can be drawn from the results of a loading test unless the subsoil is homogeneous to a great depth.

The same conclusion holds true for pile foundations. The examples quoted in the report refer exclusively to "floating pile foundations," in which the load is carried by skin friction only. Quite recently, the writer had an opportunity to make a similar observation for conical piles, 20 ft long, the points of which were driven to a depth of 2 or 3 ft into a stratum of firm gravel. The walls of the building rested on continuous footings suported by the piles. Each pile had to carry a load of 25 tons. The piles were arranged in double rows, with an average distance of 3 ft from center to center. One pile near the center of the building was selected for making a loading test. After the footings were built a bench-mark was established immediately above the test pile for the purpose of measuring its settlement as a member of the foundation. In Fig. 61 the upper curve represents the results of the loading test,

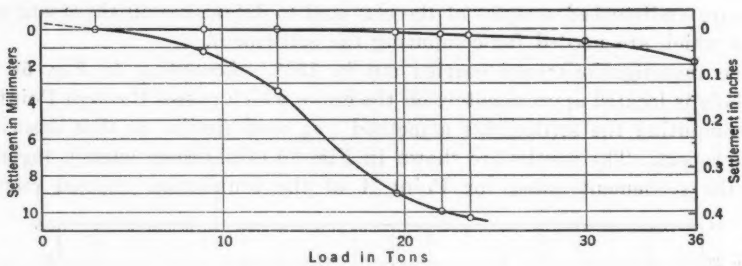

Fig. 61

and the lower curve, the results of the observations made during construction. The diagram shows that the settlement of the foundation amounted to more than eight times the settlement observed during the loading test.

Considering the validity of the Boussinesq formulas illustrated by Fig. 56, the question arises as to the depth to which borings for a new structure should be made. In order to establish a general rule for estimating this depth, one must first consult one's experience as to the minimum soil pressure that is likely to produce an appreciable compression of a stratum of the type, S (Fig. 54).

For one special case on the Pacific Coast the writer was in a position to prove that the settlement of the structure (more than 12 in.) was due to the compression of a layer of clay, 30 ft thick, at an average depth of 120 ft below the surface[44]. According to Boussinesq the maximum soil pressure produced by the weight of the building was, within the clay, equal approximately to 0.3 ton per sq ft.

In the case illustrated by Fig. 58 (greatest settlement, 36 in.) the clay was at a depth from 23 to 73 ft, and the maximum soil pressure which acted on the clay was 0.4 ton per sq ft. Hence, if the underground is likely to contain layers of a fairly soft clay, the borings should be carried at least to the depth at which the soil pressure, p (Fig. 56(b)), drops below a value of approxi-

[44] "Settlement Analysis—The Backbone of Foundation Engineering," by Charles Terzaghi, M. Am. Soc. C. E., *Transactions*, World Eng. Congress, Tokyo, 1929.

mately 0.2 ton per sq. ft. The determination of this depth can easily be made by means of Fig. 62. Let $2b$ be the width of the area covered by the building; p, the average soil pressure, in tons per square foot of the area covered by the building; and t, the minimum depth to which the test borings should be carried below the base of the proposed foundation.

The abscissas of Curve A indicate the soil pressures beneath the base of the foundation for a building covering an approximately square area, and the abscissas of Curve B those for a building on an elongated rectangle. For

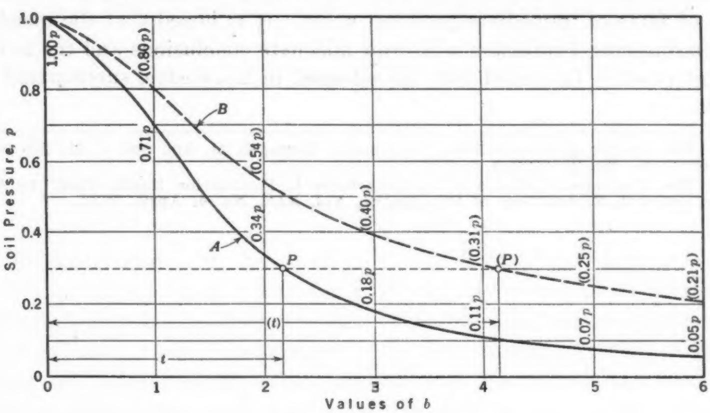

Fig. 62

a building on a raft foundation the depth, t, can be determined directly as shown in the diagram. If the building is to rest on individual or on continuous footings, two independent determinations of t should be made. The first one is identical with the foregoing procedure, whereby p should be made equal to the total load divided by the total area covered by the building (pressure, p', Fig. 56). For the second determination, it should be assumed that p denotes the pressure acting on the base of the footings, and $2b$, the width of the footings. Thus, one obtains two values for t. The greater one should be selected. In no case should the depth of the boring be less than 30 ft. A departure from the suggested procedure should be tolerated in such cases only, where there can be no doubt as to the nature of the deeper strata of the underground. It is much safer to make a crude guess as to bearing capacity on the basis of the results of adequate test borings, than to depend on the results of loading tests which, according to Fig. 56, can be misleading.

The last group of important facts concerns the effect of "remoulding" on the strength and bearing capacity of clays. The effect of "remoulding" on the stability of slopes was recognized as early as 1922 by the members of the Swedish Geotechnical Commission in Stockholm[45]. In 1927, the writer called attention to the fact that the "remoulding" connected with pile-driving is likely, in certain cases, to have a detrimental effect on the bearing

[45] Rept., Swedish Geotechnical Comm., 1914–1922, Stockholm, 1922.

capacity of the substrata[46]. Finally, in 1931, Mr. Casagrande succeeded in explaining this strange phenomenon[47] and, due to his painstaking investigations, it is now known that the "remoulding" changes almost every property of the clay, including its compressibility and permeability. For this reason, no test boring can be expected to furnish reliable information as to the character of the subsoil, unless undisturbed samples of the strata are secured. Since the report of the Committee was published the method of securing undisturbed samples has undergone rather important modifications.

In conclusion, the writer feels safe in stating that this report represents a score of facts of immediate practical value. It is hoped that the members of the Engineering Profession will draw adequate conclusions and try to modify current practice in accord with the advance in knowledge, summarized in the report.

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[46] "The Science of Foundations," by Charles Terzaghi, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol 93 (1929), p. 270.

[47] "The Structure of Clay and Its Importance in Foundation Engineering," by A. Casagrande, *Journal*, Boston Soc. of Civ. Engrs., Vol. XIX, No. 4, April, 1932.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THREE-SPAN CONTINUOUS-TRUSS RAILROAD BRIDGE, CINCINNATI, OHIO

Discussion

By J. E. Willoughby, M. Am. Soc. C. E.

J. E. Willoughby,[5] M. Am. Soc. C. E. (by letter).[5a]—The plans for, and construction of, the south river pier of the bridge described by Mr. Ballard are of much interest to those who are responsible for the reconstruction of existing bridges. Definite economies result from the preservation of existing masonry in bridge reconstruction projects, and the author has described a successful engineering solution of a specific problem for a great bridge. It is to be regretted that the requirements of the War Department prevented similar extension construction for the north river pier, with its resulting economy to the users of transport. Now that the author has proved the case with regard to the substructure of a great bridge, engineers may expect the adoption of his methods for future similar projects, thus furthering the economy of bridge reconstruction. It is a prevailing error of those in charge of reconstruction to cast away much of the existing works instead of incorporating them into the rebuilt structure. For railway bridges, the elaborate accounting system prescribed by public authority adds to the waste.

For many years, the writer has endeavored to preserve the existing masonry in the bridge reconstruction projects of railway bridges. While those projects have not been of magnitude much money has been saved to the owners of the bridges. A late project was the strengthening of the cast-iron cylinders that formed a pier for a railway bridge over the Cape Fear River, near Wilmington, N. C. The pier was built in 1868–69, and consisted of two cast-iron cylinders, each 96 in. in diameter, founded on marl 50 ft below mean low tide. The pier supported one end of a through truss fixed span, 216 ft in length, and served as a rest pier for a rolling-lift draw span,

Note.—This paper by Wilson T. Ballard, M. Am. Soc. C. E., was presented at the meeting of the Construction Division, New York, N. Y., on January 22, 1931, and published in April, 1933 *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[5] Chf. Engr., A. C. L. R. R., Wilmington, N. C.

[5a] Received by the Secretary May 3, 1933.

118 ft in length. The steps to strengthen the pier and to give it added bearing area on the marl were, to:

(1) Clear, by dredging, the accumulation of riprap, logs, silt, and sand from around both cylinders, the sunken logs being removed with locomotive crane;

(2) Construct about the two cylinders a shell of $\frac{3}{8}$-in. steel plate, assembled in the field, section by section, above mean tide, and then lowered;

(3) Pump and jet from the interior of the shell, and around the outside, so as to free the shell for further lowering, until the bottom of the river, 43 ft below mean tide, was reached by the lower edge of the shell;

(4) Remove, by pumping, all loose sand and débris from the marl in the interior of the shell, when the lower edge of the shell has reached that depth;

(5) Place a seal of concrete through the tremie, and add concrete above the seal sufficient to prevent displacement and distortion of the shell by the bouyancy and pressure of the water outside; and

(6) Unwater the shell and complete the concrete encasement of the two cylinders.

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The Constitution provides that the Board of Direction shall elect or reject all applicants for *Admission* or for *Transfer,* and, in order to determine justly the eligibility of each candidate, the Board must depend largely upon the Membership for information.

This list is issued to members in every grade for the purpose of securing all such available information, and every member is urged to scan carefully each monthly list of candidates and to furnish the Board with data in regard to any applicant which may aid in determining his eligibility. It is the *Duty* of all *Members* to the *Profession* to assist the *Board* in this manner.

It is especially urged, in communications concerning applicants, that a *Definite Recommendation as to the Proper Grading in Each Case* be given, inasmuch as the grading must be based upon the opinions of those who know the applicant personally, as well as upon the nature and extent of his professional experience. If facts exist derogatory to the personal character or to the professional reputation of an applicant, they should be promptly communicated to the Board. *Communications Relating to Applicants are considered by the Board as Strictly Confidential.*

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Grade	General Requirement	Age	Length of Active Practice	Responsible charge of work
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Junior	Qualified for sub-professional work	20 years†	4 years*	
Affiliate	Qualified by scientific acquirements or practical experience to co-operate with engineers	35 years	12 years*	5 years of important work
Fellow	Contributor to the permanent funds of the Society			

*** Graduation from a school of engineering of recognized reputation is equivalent to 4 years of active practice.**

† Membership ceases at age of 33 unless transferred to higher grade.

The fact that applicants give the names of certain members as references does not necessarily mean that such members endorse.

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BASTA, RUDOLPH FRANCIS, Council Bluffs, Iowa. (Age 23.) Timekeeper and Instrumentman, Ryan Constr. Co. of Omaha. Refers to M. I. Evinger, H. J. Kesner, C. E. Mickey.

BAUKNIGHT, WILFRED, Rosiclare, Ill. (Age 22.) Inspector, U. S. Engr. Dept. Refers to R. C. Johnson, W. E. Rowe.

de RYSS, EMIL, Scarsdale, N. Y. (Age 55.) Cons. Engr., Rapid Transit Subway Constr. Co., New York City. Refers to C. E. Carpenter, F. W. Gardiner, W. C. McNaughton, J. H. Myers, H. G. Pegram, W. F. Reeves, R. Ridgway.

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FLACK, WILLIAM REITZ, Medina, Ohio. (Age 32.) Res. Engr., Ohio State Dept. of Highways. Refers to W. W. Anderson, W. K. Hatt, F. L. Plummer, G. P. Springer, W. A. Stinchcomb.

FOX, FREDERICK JAY, Bronx, N. Y. (Age 39.) Secy., Arthur I. Kraft, Inc., and Pres., Lehigh Pile & Foundation Corporation. Refers to E. A. Byrne, A. Dick, J. Feld, J. J. Murphy, B. Schwerin, A. V. Sielke.

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ROHLWING, ANSON WILLIAM, Jun., Lousville, Ky. (Elected Oct. 1, 1926.) (Age 32.) Field Engr., Portland Cement Association. Refers to W. D. Beisell, W. R. Britton, F. C. Dugan, E. M. Fleming, G. R. Harr, N. M. Stineman, W. J. Titus.

SALAGE, DAVID, Jun., Flushing, N. Y. (Elected July 15, 1929.) (Age 32.) Asst. Engr., Grade 4, Dept. of Water Supply, New York City. Refers to W. W. Brush, H. R. Codwise, F. E. Foss, C. F. Giraud, J. Goodman, M. H. Van Buren, J. P. J. Williams.

SPELMAN, MALCOLM STEWART, Jun., Rockville Centre, N. Y. (Elected Feb. 25, 1924.) (Age 32.) Contractor, Point Lookout, N. Y. Refers to D. Bonner, E. W. Bowden, A. Brahdy, C. Carswell, H. V. Penn, E. W. Stearns, J. E. Tonnelier.

THOMPSON, PERCY EDWARD, Jun., Glendale, Cal. (Elected March 30, 1931.) (Age 32.) With Bridge Div., Los Angeles County Road Dept. Refers to W. D. Armstrong, E. A. Burt, O. F. Cooley, M. M. Falk, G. W. Jones, H. F. Pope, H. E. Warrington.

TOTH, ALEXANDER STEPHEN, Jun., Philadelphia, Pa. (Elected Jan. 16, 1928.) (Age 32.) Refers to J. J. Costa, A. A. Dedouloff, R. Kleppe, E. P. Leclercq, J. A. Wahler, R. R. Wiggins, W. H. Yates.

WILKIN, INCREASE CROSBY JORDAN, Jun., Gardiner, N. Y. (Elected Oct. 21, 1924.) (Age 32.) Land Surveyor. Refers to O. H. Bundy, G. C. George, J. F. Loughran, R. T. Retz, H. C. Sandbeck, W. A. Schuerman.

The Board of Direction will consider the applications in this list not less than thirty days after the date of issue.